Stability Analysis of Fluid Loaded or Supported Shell Structures

M. Haßler, K. Schweizerhof
Universität Karlsruhe, Institut für Mechanik

24. Mai 2006

Institut für Mechanik
Kaiserstr. 12, Geb. 20.30
76128 Karlsruhe
Tel.: +49 (0) 721/ 608-2071
Fax: +49 (0) 721/ 608-7990
E-Mail: ifm@uni-karlsruhe.de
www.ifm.uni-karlsruhe.de

Stability Analysis of Fluid Loaded or Supported Shell Structures

M. Haßler, K. Schweizerhof

May 24, 2006

Abstract

Fluid or gas loadings on thin shell or membrane structures under external loadings (e.g. in oil tanks or gas supported beams) may have a major infuence on the structural stability behavior. The goal of this approach, which is based on the works of [2], [3] and [4], is to present some investigation of the influence of such a gas or fluid support (without actually discretizing the fluid/gas) on the eigenvalues and eigenmodes of the stiffness matrix of shell or membrane-like structures undergoing large displacements to allow conclusions concerning stability. For this purpose an efficient algorithm is derived, which benefits from the dyadic rank updates of the stiffness matrix due to volume dependence [1], [2] of the fluid/gas loading.

1 Modal Analysis

Depending on the load case the volume dependence of the fluid or gas pressure leads to several rank updates of the global stiffness matrix $\boldsymbol{K}$ (see [2], [3]). For reasons of simplicity the special case of a single chamber only loaded with an incompressible fluid will serve as an example. The global system matrix $\boldsymbol{A}$ then becomes

$$\boldsymbol{A} = \underbrace{\boldsymbol{K}}_{\text{stiffness part}} + \underbrace{\gamma_t \boldsymbol{a}\boldsymbol{a}^T}_{\text{volume coupling part}} , \tag{1.1}$$

This dyadic rank update with the volume coupling vector $\boldsymbol{a}$ and a proportional factor γ_t results in a fully occupied part of the system matrix $\boldsymbol{A}$, which may lead to numerical difficulties during the modal decomposition process. To bypass the standard procedure for computing the modal matrix $\boldsymbol{\Phi}$ and the eigenvalues χ of an almost fully populated system matrix $\boldsymbol{A}$ the modal matrix $\boldsymbol{\Phi}$ is split up into a modal factor matrix $\boldsymbol{\Xi}$ and the modal matrix $\boldsymbol{\Psi}$, which contains all eigenvectors $\boldsymbol{\psi}_i$ of the uncoupled stiffness matrix $\boldsymbol{K}$. Thus $\boldsymbol{\Phi}$ can be written as:

$$\boldsymbol{\Phi} = \boldsymbol{\Psi}\boldsymbol{\Xi} = \begin{pmatrix} | & | & & | & & | \\ \boldsymbol{\psi}_1 & \boldsymbol{\psi}_2 & \dots & \boldsymbol{\psi}_i & \dots & \boldsymbol{\psi}_n \\ | & | & & | & & | \end{pmatrix} \begin{pmatrix} | & | & & | & & | \\ \boldsymbol{\xi}_1 & \boldsymbol{\xi}_2 & \dots & \boldsymbol{\xi}_i & \dots & \boldsymbol{\xi}_n \\ | & | & & | & & | \end{pmatrix} \tag{1.2}$$

Introducing the new matrix

$$\boldsymbol{A}^\star = \boldsymbol{\Psi}^T \boldsymbol{A} \boldsymbol{\Psi} = \boldsymbol{\Psi}^T \left(\boldsymbol{K} + \gamma_t \boldsymbol{a}\boldsymbol{a}^T \right) \boldsymbol{\Psi} \ . \tag{1.3}$$

and after some rearranging a modified form of the eigenvalue problem $\boldsymbol{A}\boldsymbol{\Phi} = \boldsymbol{\Phi}\boldsymbol{X}$ can be given with the columns $\boldsymbol{\xi}_i$ of the modal factor matrix $\boldsymbol{\Xi}$ and the eigenvalues χ_i as:

$$\boldsymbol{A}^\star \boldsymbol{\xi}_i = \chi_i \ \boldsymbol{\xi}_i \ . \tag{1.4}$$

Substituting (1.3) into (1.4) gives us after some reordering

$$\left(\boldsymbol{\Psi}^{\boldsymbol{T}}\left[\boldsymbol{K}+\gamma_t \boldsymbol{a}\boldsymbol{a}^T\right]\boldsymbol{\Psi}-\chi_i \boldsymbol{I}\right)\boldsymbol{\xi}_i=\mathbf{0}\ . \tag{1.5}$$

Using the spectral matrix $\boldsymbol{\Lambda}$ (with its eigenvalues λ_i) of $\boldsymbol{K}$ along with the modified coupling vector $\bar{\boldsymbol{a}}=\boldsymbol{\Psi}^T\boldsymbol{a}$ yields

$$\left(\boldsymbol{\Lambda}+\gamma_t\bar{\boldsymbol{a}}\bar{\boldsymbol{a}}^T-\chi_i\boldsymbol{I}\right)\boldsymbol{\xi}_i=\mathbf{0}\ . \tag{1.6}$$

1.1 Eigenvalue extraction

For this homogeneous set of equations non-trivial solutions $\boldsymbol{\xi}_i \neq \mathbf{0}$ do exist, if the determinant of the coefficient matrix $\left(\boldsymbol{\Lambda}+\bar{\boldsymbol{a}}\bar{\boldsymbol{a}}^T-\chi_i\boldsymbol{I}\right)$ vanishes. As the new eigenvalues are different from the old ones, $\chi_i \neq \lambda_i$, the unknown eigenvalues χ_i can be extracted as solutions of the characteristical polynomial $p(\chi)$:

$$p(\chi)=1+\gamma_t\sum_{j=1}^{n}\frac{\bar{a}_j\bar{a}_j}{\lambda_j-\chi}=0\ , \qquad \text{with} \qquad \begin{pmatrix} \chi_i>\lambda_i & \text{for} & \gamma_t>0 \\ \chi_i<\lambda_i & \text{for} & \gamma_t<0 \end{pmatrix} \tag{1.7}$$

Thus the new eigenvalues χ_i will increase or decrease depending on the sign of the factor γ_t. As already mentioned the polynomial $p(\chi)$ is strictly monotone between the poles λ_j, therefore an efficient method to localize the zeroes in $p(\chi)$ can be found by the bisection method.

1.2 Computation of Eigenvectors

For the computation of the corresponding eigenvectors equation (1.6) is considered again, focusing on the column $\boldsymbol{\xi}_i$ of the modal factor matrix $\boldsymbol{\Xi}$. To eliminate the implicit form of $\boldsymbol{\xi}_i$ it can be normalized by its length, leading to

$$\boldsymbol{\xi}_i=-\frac{\gamma_t\left(\boldsymbol{\Lambda}-\chi_i\boldsymbol{I}\right)^{-1}\bar{\boldsymbol{a}}\ \left(\bar{\boldsymbol{a}}^T\cdot\boldsymbol{\xi}_i\right)}{||\gamma_t\left(\boldsymbol{\Lambda}-\chi_i\boldsymbol{I}\right)^{-1}\bar{\boldsymbol{a}}\ \left(\bar{\boldsymbol{a}}^T\cdot\boldsymbol{\xi}_i\right)||}=-\frac{\left(\boldsymbol{\Lambda}-\chi_i\boldsymbol{I}\right)^{-1}\bar{\boldsymbol{a}}}{||\left(\boldsymbol{\Lambda}-\chi_i\boldsymbol{I}\right)^{-1}\bar{\boldsymbol{a}}\ ||} \tag{1.8}$$

as the i^{th} column of $\boldsymbol{\Xi}$. With the multiplicative split (1.2) the transformed modal matrix can then be computed.

2 Numerical Example

To show the effect of the fluid support on the stability behavior the development of the eigenvalues of steel cylinder ($r = 20m$, $h = 40m$, $r/t = 1000$) under axial loading σ will be monitored (see figures 2.1 and 2.2). Increasing the axial loading σ on the empty cylinder (a) ends up at an critical buckling load of $\sigma_{max} = 7.1 \cdot 10^{-5} N/mm^2$. The fluid filling of cylinder (b) leads to an increase of the eigenvalues (mainly due to hydrostatic pretensioning) and the subsequent axial laoding leads to a critical buckling load of $\sigma_{max} = 8.3 \cdot 10^{-5} N/mm^2$, which is about 16% higher than for the empty cylinder. Further on the volume coupling had only small effects on the eigenvalues of the cylinder because the deformations in this example are relatively small. Other examples of soft structures undergoing large deformations showed a clear stiffening due to volume dependence. A stronger effect of the volume coupling on stiffer structures, as e.g. the steel cylinder, is expected for a cylinder completely filled with a compressible heavy fluid, which will be part of further investigations.

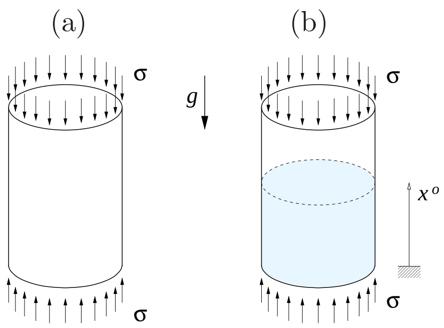

Fig. 2.1: Empty (a) and fluid filled cylinder (b) under axial loading σ

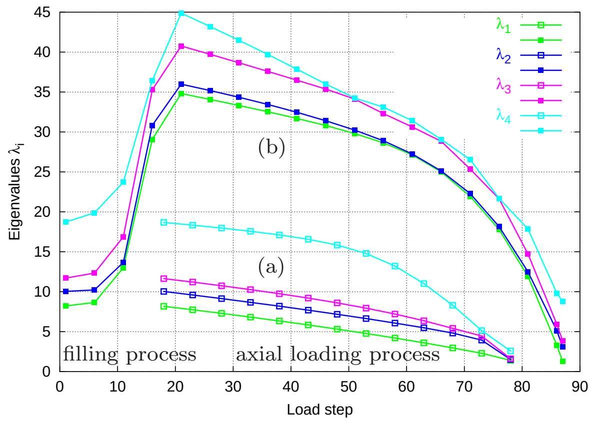

Fig. 2.2: Development of eigenvalues during loading process

References

[1] J. Bonet, R. D. Wood, J. Mahaney, P. Heywood, *Finite element analysis of air supported membrane structures*. Computer Methods in Applied Mechanics and Engineering, **190**, 579–595 (2000).

[2] T. Rumpel and K. Schweizerhof, *Volume-dependent pressure loading and its influence on the stability of structures*. International Journal for Numerical Methods in Engineering, **56**, 211–238 (2003).

[3] T. Rumpel and K. Schweizerhof, *Hydrostatic fluid loading in non-linear finite element analysis*. International Journal for Numerical Methods in Engineering, **59**, 849–870 (2004).

[4] T. Rumpel, K. Schweizerhof and M. Haßler, *Efficient Finite Element Modelling and Simulation of Gas and Fluid Supported Membrane and Shell Structures*. Recent Advances in Textile Membranes and Inflatable Structures, E. Onate, B. Kröplin (eds.) (2004).

[5] K. Schweizerhof and E. Ramm, *Displacemant dependent pressure loads in non-linear finite element analyses*. Computers & Structures, **18**, 1099–1114 (1984).