On the numerical analysis of a carillon bell using LS-DYNA with a specific view on model validation

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Abstract:

In this contribution we focus on the validation of a numerical model for a carillon bell. Both, the realization of the numerical model and the experimental set up are described. The results of the experimental and numerical investigations are used to estimate the quality of the numerical model. For this purpose, the eigenfrequencies of the experimental and numerical models are compared and the orthogonality of the eigenmodes is evaluated via a modal assurance criterion.

Keywords:

bell, FEM, experimental modal analysis, model validation

1 Introduction

Bells are used over several millenniums both for profane purposes as e.g. signaling imminent danger or as musical instrument and for religious purposes like church bells. With the extending Christendom the amount of bells increased and the previous small forms of bells without ornaments was replaced by bigger bells with artful ornaments. The form also changes over the centuries. Starting with half spheres or egg forms, in the European area also beehive forms, the gothic bell form is now the dominant form [2].
Up to now the production of a bell starts with the choice of a so called bell rib. The bell rib is a kind of stencil of the inner and outer surface of the bell. The form of the stencil is similar to the vertical cross section of the bell. It is similar because the original stencils and the cross section of the later poured bell differ due to thermal stretching and restraint stresses. With this bell ribs the cast is build up from clay and masonry. After the casting, the bell is removed by destroying the cast. In this way every bell is unique. Further information on the production process can be found in [2],[8].
From time to time, some of the new produced bells failed the appraisal of a bell expert, because the sound has an unwanted beat. This beat is a result of very close eigenfrequencies, which arise typically in nearly rotationally symmetric structures. If the distance of these close eigenfrequencies is over a subjective limit, the sound can be recognized as dissonant. For these reasons, strong efforts have been made to explore the phenomena of so called double or spinning modes and the influence of ornaments, which disturb the exact rotational symmetry. These kind of problems can be very well investigated by numerical analysis, mainly modal decomposition, to predict the influence of ornaments or of slight deviations from rotational symmetry [3], [4], [10], [11], [12].
The numerical analysis needs a carefully designed numerical model. In particular, attention should be paid to the choice of elements, materials and the solution algorithm. A validation of a model must be enforced to ensure the quality of results. Typical methods for the validation of vibrating systems are the comparison of eigenfrequencies and eigenmodes from a numerical and experimental investigation. In this study the main focus lies on the validation process of a numerical model of a carillon bell. The carillon bell was provided by courtesy of Rudolf Perner GmbH & Co., Passau.
The validated models can be used to investigate the influence of ornaments as well as for investigations of material fatigue, or of the production process. Investigations for new forms and different sounds can be made quasi virtually using the modern instruments of numerical analysis.

2 Numerical investigations

To predict the physical behavior of the carillon bell, especially the vibrations, a suitable mathematical model has to be formulated and finally numerically solved. The results must be interpreted in comparison to experimental or real world observations. For this study the Finite Element Method is chosen to accomplish the analysis of the bell vibrations. Using this method, first a spatial discretization has to be conducted, adequate material parameters have to be defined and a dynamic eigenfrequency analysis must be preformed.

2.1 Numerical model

The carillon bell was poured by Rudolf Perner GmbH & Co., Passau. The bell has a maximum diameter of approximately 25cm and a height from approximately 30cm. The bell is made out of bronze and has a little ornament which shows the logo of the producer. To create a numerical model, the bell geometry was measured in a first step. The outer surface was measured directly in a customized measure construction (see Figure 1). To measure the inner surface, a plaster cast has been produced and measured in the above-mentioned construction.
The measurement points generated by this procedure are taken as measuring points for the experimental investigation as well as for the nodes of spatial discretization of the numerical model. This ensures the consistency of the results taken from the experimental measurements and the numerical results. The discretized model is shown in Figure 2.
The model considered here contains 2524 solid elements with hexahedral shape and trilinear ansatz functions. For other discretizations and mesh studies we refer to [12]. A one point integrated element formulation with a physical hourglass stabilization is used for both the implicit eigenfrequency analysis and the transient calculations. The one point integrated element formulation in combination with the physical hourglass stabilization is well behaved both in incompressible regimes and bending dominated problems. Further details on the physical stabilization can be found in [5], [6] and [1].
We chose a linear elastic material model, since no plastic deformation appears in the experimental study. It must be noted that in the case of an analysis with an impact by a clapper or fatigue

investigations, the material model must be adapted in the way that the typical phenomena shown by the material taken into account by the material formulation. In the actual analysis, the density was set to 8380 kg/m³ and the elastic modulus equals 96500 N/mm². The elastic modulus was determined through a parameter study. The eigenfrequencies from the experimental and numerical investigations are compared until they reach a predefined threshold [11]. The elastic modulus is as free parameter taken from a realistic range of values in this analysis.
Both kinds of analysis, the implicit modal decomposition and the explicit in the time domain, was performed by LS-DYNA [9].

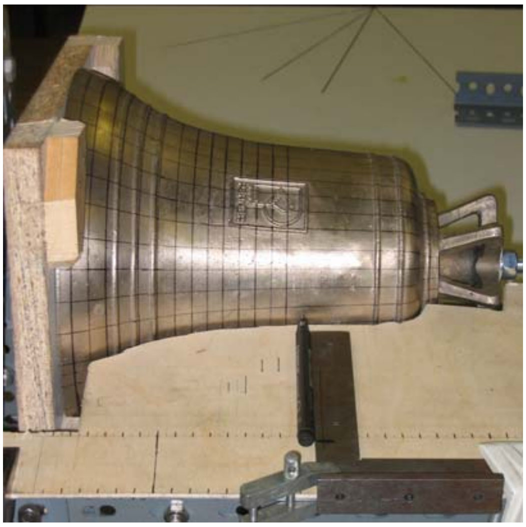
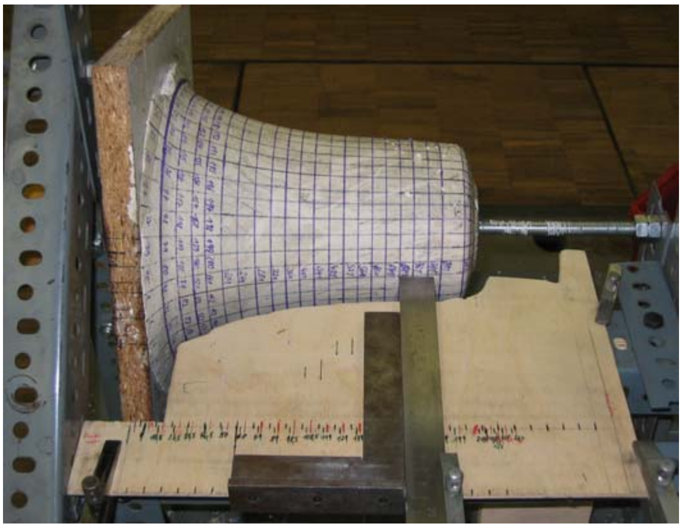



Figure 1. *Customized measure construction, left: capture of the outer surface of the bell, right: capture of the inner surface via the plaster cast*

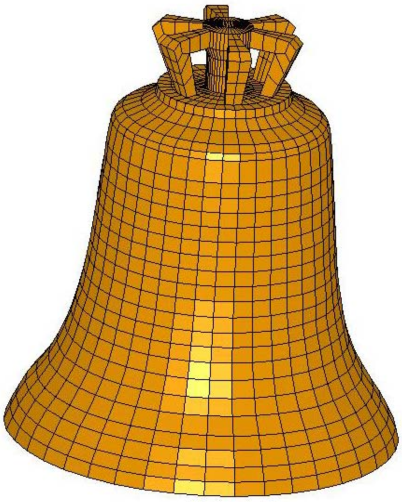


Figure 2. *Discretized model of the carillon bell*

To validate the numerical model a dynamic eigenfrequency analysis was performed (see Table 1). The eigenfrequencies and eigenmodes provide the basis for the later model check procedure.

3 Experimental Modal Analysis

The second part of the model validation and the subsequent model check process needs an experimental investigation to compare measured results to numerical results. An experimental setup was created to acquire the structural response on the mesh nodes enforced by modal hammer impact. The signal processing and post processing has been performed using the software tool I-DEAS Test [7].

3.1 Experimental setup

The grid points of the geometrical measurement are used to place the accelerometers. Because the grid points serve as both nodes in the discretization and measuring points in the experimental setup the consistency of the results is preserved. The modal hammer was fixed by a carrier. This ensures that the location of the impact point is constant for all measurements. The hammer force has been controlled via constant initial elevation in order to guarantee similar impact velocities within the entire data acquisition process. Four uniaxial accelerometers were “traveling” over the outer surface grid of the bell and data at 199 grid points was collected. A very small mass ratio between sensors and specimen (bell) helps to avoid mass load effects due to the traveling accelerometers.

3.2 Post processing of the experimental measurements

As mentioned before, the post processing was performed by the software tool I-DEAS. In Figure 3 a typical result of the experimental investigation is given, showing both the amplitude and the phase information of the complex frequency response function (FRF):

$$H_{ij}(\omega) = \frac{A_i(\omega)}{F_j(\omega)} \quad (1)$$

A_i denotes the spectrum of the acceleration response captured at test point i and F_j is the spectrum of the impact force at the driving point j. The maximum amplitudes of the frequency response function (see Figure 3) are connected with the eigenfrequencies of the structure. Furthermore, vanishing values of the real part of the frequency response function $Re(H_{ij})$ indicate the eigenfrequencies. Modal damping parameters are estimated by the frequency response functions using a single degree of freedom method.

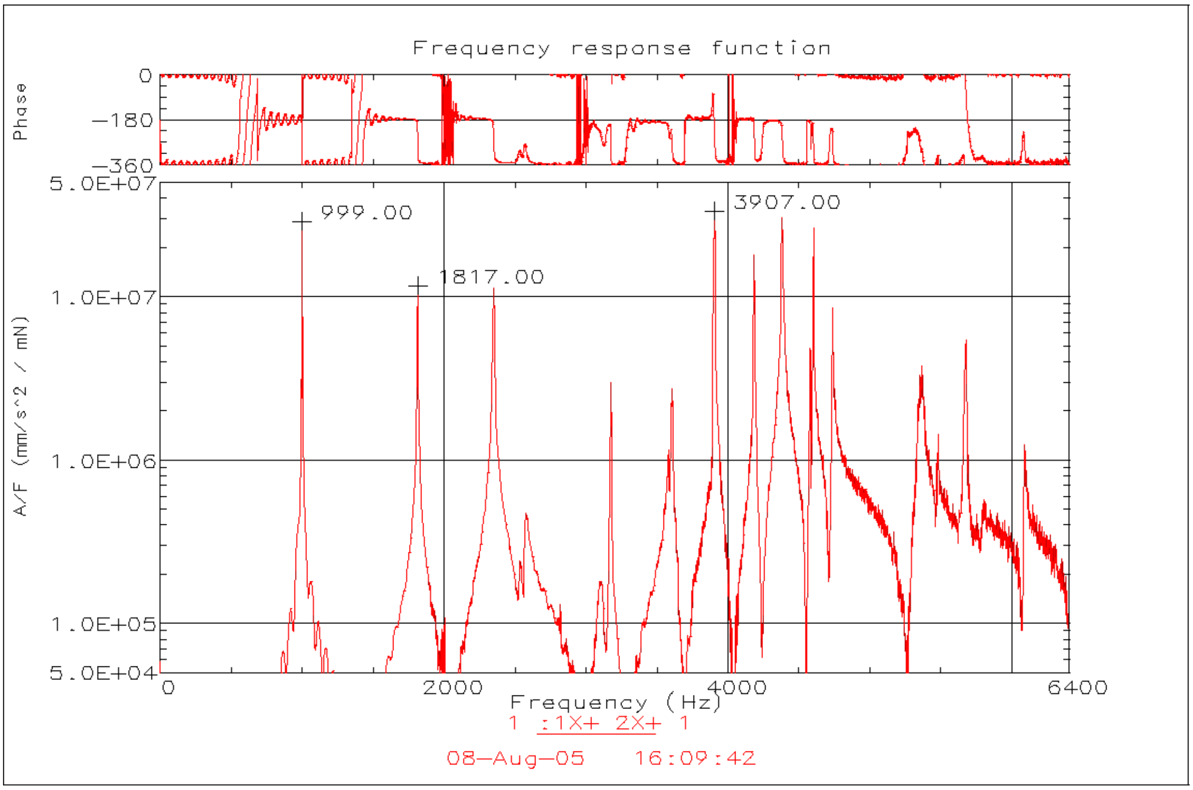


Figure 3. *Example of an experimentally obtained frequency response function for the identification of modal parameters: eigenfrequencies, damping.*

4 Validation of the bell model

To achieve a good correlation of the model with the experimental results two main parameters of uncertainty were identified that need to be determined in the model check process. Since the geometry of the bell is measured with sufficient accuracy, only a limited uncertainty occurs concerning these data. Thus, the material properties as given by the elastic modulus and the mass density are the parameters to be adjusted. The mean density is correlated to the overall weight of the bell, which is a fixed value. A moderate way is to assume the bell as homogeneous solid which means a constant allocation of the mass density over the whole spatial domain. Finally, the only remaining parameter to be adjusted within a model updating process is the elastic modulus.
In case the model could not be validated or the discrepancies were too large more details must be taken into account. For example after the casting process the material allocation can be disturbed by voids and shrink holes. If this happens, both the modulus and the density are dependent on their location at the bell. This kind of problem can be solved by introducing these parameters as spatial functions and seeking for the best fit using a global optimization procedure.
The model check procedure contains three steps. First, the deviation of selected eigenfrequencies from the computation and experiment is checked and when necessary minimized by adjusting the elastic modulus. For this purpose the frequency criterion

$$R_f = \sum_i \left(\frac{f_{i,num}^2 - f_{i,exp}^2}{f_{i,exp}^2} \right)^2 \rightarrow \mathrm{MIN} \tag{2}$$

is applied as the objective function. In a second step, the obtained eigenmodes are compared visually. In the third step, the model quality is evaluated by a modal assurance criterion (MAC).

$$\mathrm{MAC}(i,j) := \frac{\left(e_i^T e_j\right)^2}{\left(e_i^T e_i\right)\left(e_j^T e_j\right)} \qquad e_k \ldots k\text{-th eigenmode of the structure} \tag{3}$$

It expresses how similar the shapes are by calculating the coherence between them. Where the visual mode shape comparison gives visual feedback through animations, the MAC returns a value between 0 and 1 for each pair of eigenmodes. With the help of this criterion the orthogonality condition between the experimental and the numerical eigenmodes can be assessed. Furthermore, the criterion yields information about the quality of the spatial discretization to be chosen.

4.1 Eigenfrequencies and eigenmodes

First, the eigenfrequencies of the experimental investigations and the numerical analysis are compared. In Table 1 the so called “sound creating” frequencies are compared. “Sound creating” in this context means, that a bell sound is mainly influenced by these frequencies.

Sound name	experimental frequency [Hz]	numerical frequency [Hz]	Deviation [%]
Octave (below, first)	999	975	2,5
Octave (below, second)	999	978	2,1
Prime (first)	1814	1872	3,2
Prime (second)	1817	1880	3,5
Third (first)	2351	2350	0,0
Third (second)	2351	2353	0,1
Quint (first)	3176	3150	0,8
Quint (second)	3176	3159	0,5
Octave (above, first)	3907	3944	0,9
Octave (above, second)	3907	3946	1,0
12th (first)	5677	5793	2,0
12th (second)	5668	5802	2,4

Table 1. *Comparison of experimental and numerical acquired eigenfrequencies (selection)*

The experimental and numerical eigenfrequencies show a good agreement. Finally, a maximum deviation of 3,5 % is obtained. Thus a model updating appears not to be necessary concerning frequencies. The bell under consideration is a real structure and shows therefore no exact rotational symmetry. Hence, no exact double eigenfrequencies exist. There are some pairs of neighboring eigenfrequencies which are close together and have a similar visual shape of the eigenmode. Therefore in Table 1 always two values are belonging to one partial.

Secondly, the eigenmodes are visually compared. In Table 2 five typical partials are presented. In the pictures, the darker areas are regions showing smaller deflections; the lighter ones identify areas with a distinct deflection. The experimentally and numerically calculated modes display good agreement. It should be mentioned that a slight rotation between the forms can be observed. This rotation along the central symmetry line can be recognized both between the experimental and numerical modes and the two eigenmodes with close eigenfrequencies due to the non-perfect rotational symmetry.

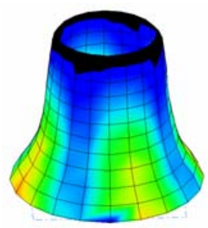
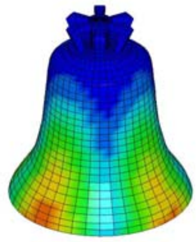
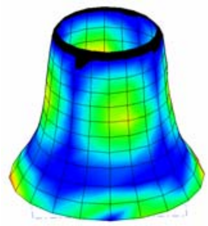
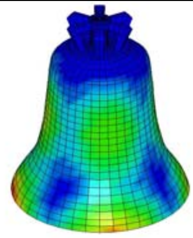
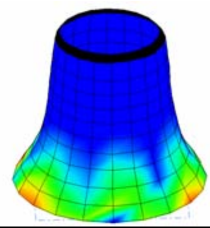
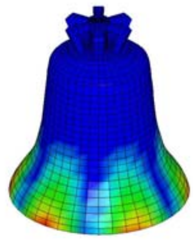
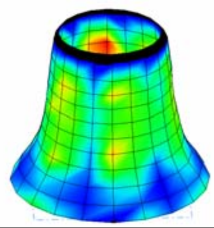
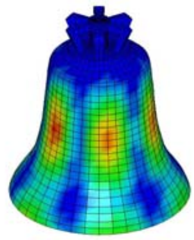
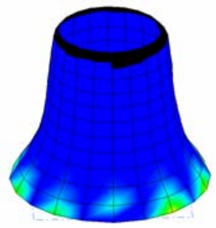
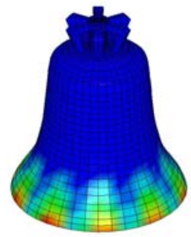
tone and frequency experimental / numerical	Experimentally obtained eigenmode	Numerically obtained eigenmode
Oktave (below) 999 Hz / 975 Hz		
Prime (first) 1814 Hz / 1872 Hz		
Third (first) 2351 Hz / 2350 Hz		
Quint (first) 3176 Hz / 3150 Hz		
12th (first) 5668 Hz / 5802 Hz		

Table 2. *Selected experimentally and numerically obtained eigenmodes*

4.2 Model validation

In a third step, a model validation is performed by the numerical and experimental modal shapes. Although the experimental shapes usually have far fewer DOFs than the numerical shapes, both shapes can be compared at all DOFs that are common to both of them. One method has been extensively used for quantitatively comparing mode shapes - the modal assurance criterion (MAC).
In this method, the deviations between the experimental and the numerical eigenmodes are considered by the so-called MAC matrix. Due to the condition of orthogonality resp. collinearity between the eigenmodes, the MAC matrix is generally expected to be a diagonal matrix. In case of distinct deviations, the numerical model should be improved or the spatial discretization of the experimental investigations must be refined – the so called model updating.
The values of the MAC matrix are measures of the angles between two eigenmodes. If a pair of eigenmodes is collinear, the MAC equals one. If the eigenmodes are orthogonal, the value is zero. Therefore a combination of eigenmodes can be checked from the viewpoint of their dependencies. In this study, three MAC matrices are calculated. The experimental and numerical determinend eigenmodes are compared to themselves (auto MAC matrix) and against each other.
It must be noted that the compared frequencies must belong to the same mode. The experimental eigenfrequencies and the computational ones may not be necessarily in the same order. So one has to make sure that the frequencies compared must belong to the same mode. The MAC value provides one possibility to test this.
The MAC matrix of the experimental eigenmodes is shown in Figure 4. Here, the MAC values are calculated by correlating two experimentally gained eigenmodes. The first twenty eigenmodes of the structure are considered. It is noticeable that the second and third eigenmode are slightly coupled. These two modes are belonging to so-called “spinning modes”. They appear in the case of a non-perfect rotational symmetry and can be identified by eigenfrequencies which are close together. Moreover, the neighboring eigenfrequencies show similar eigenmodes which are shifted by an arbitrary angle around the symmetry axis. In the considered case other spinning modes are obtained for the eigenmodes 13 to 18 and 19, 20. The MAC value between two closely spaced structural modes will, in general, show a lower correlation.

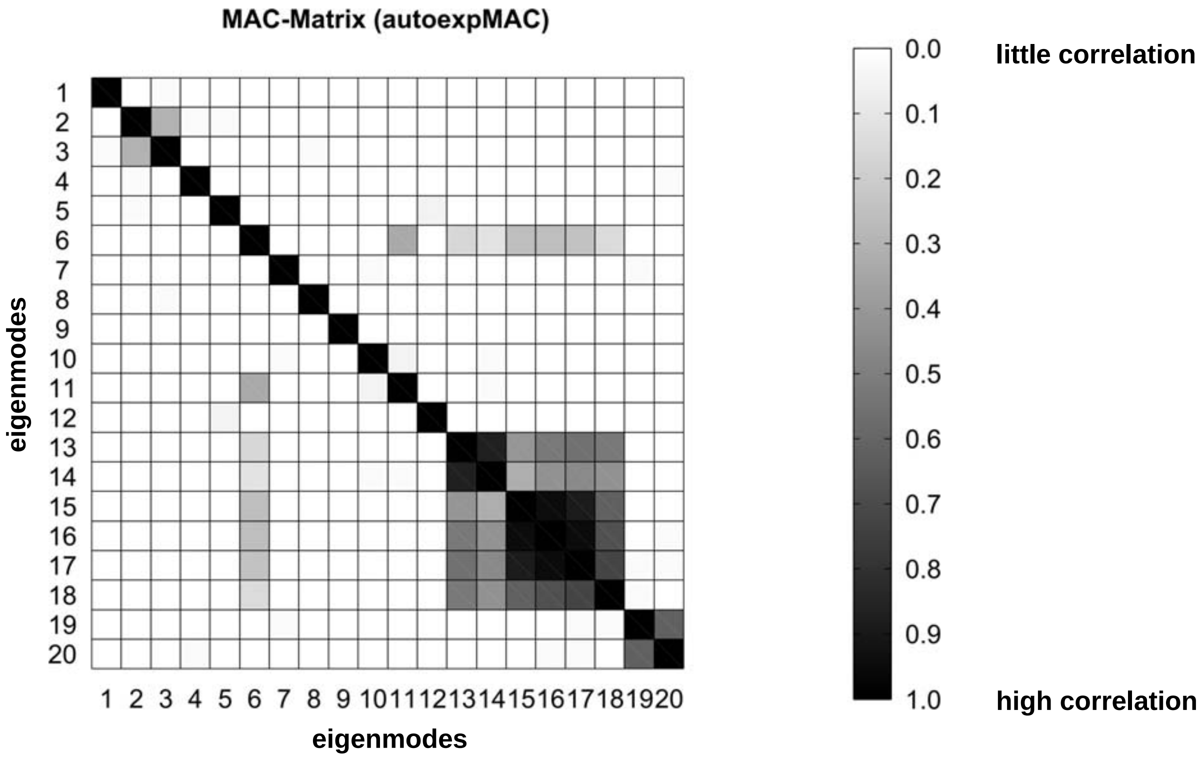


Figure 4. Auto MAC matrix of experimental eigenmodes

A special case is eigenmode 6, which is slightly coupled to several other eigenmodes. For this reason, it might be possible that the value of the related eigenfrequency of mode 6 is slightly different from the

actual eigenfrequency. Hence, the captured eigenmode may not represent a structural mode. A closer look to the phase-resonance-criteria would help to prove this conjecture, but was not performed.
A second MAC-matrix was generated from the numerical results. The MAC matrix calculated from the numerical results must be close to the identity matrix if every degree of freedom is considered. To compare the numerically determined eigenmodes to the experimental modes, both modes may only contain the same degree of freedom at the same spatial position at every component. The spatial positions chosen are the experimental test-points. The intent to produce nevertheless a MAC matrix is to check if the eigenmodes build up from the reduced database are still orthogonal. The numerical Auto-MAC matrix which is not shown here explicitly appears even with the reduced database as an identity matrix.
The third MAC matrix to be considered is the so-called Correlation MAC matrix or Cross MAC matrix. Here the eigenmodes determined from the experimental results are compared to the eigenmodes from the numerical investigation. In Figure 5 the correlation MAC matrix is displayed. For this matrix only nine eigenmodes are selected. These eigenmodes are belonging to the eigenfrequencies which are the so called “sound building frequencies” of the bell. The couples of eigenmodes in row two and three are the spinning modes of the prime.
As expected, the results reveal a matrix which is close to a diagonal one. The MAC values are ranging from 26% to 65%. The discrepancy to the expected values of approximately 80% to 90% can be anticipated by having a closer look at Table 2. The orientation node – as given by the impact point where the impulse is introduced during the experimental investigations – is the grid point in the middle at the lower end of the bell. Visually it can be proven, that the eigenmodes of the experimental and numerical investigation, and therefore the eigenmodes, are shifted around the conceived symmetry axis by a small angle. The solution of this problem requires testing again with a small rotation of either the numerical or the experimental results, which is not done yet.

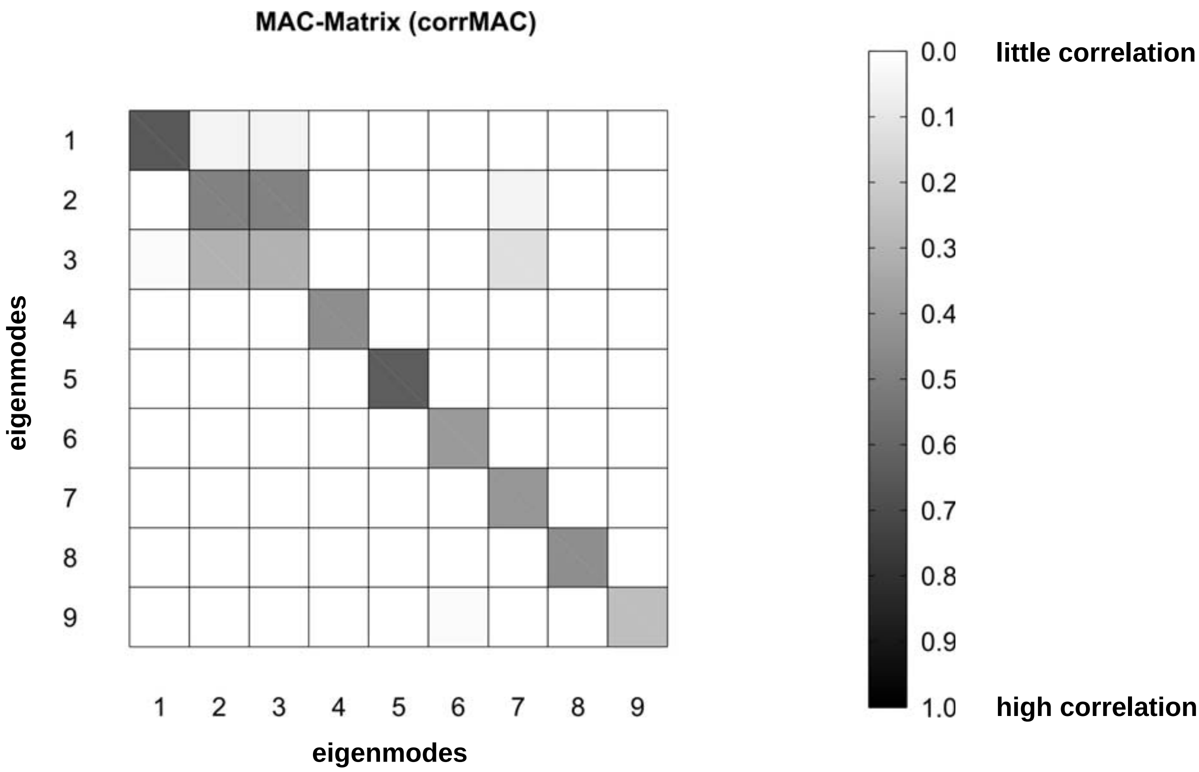


Figure 5. *Correlation MAC matrix*

5 Summary and conclusions

This study outlines the validation process of a numerical model for the vibrations of a carillon bell. As has been shown the quality of the numerical model can be determined via the eigenfrequencies and eigenmodes. In case of geometries which are close to rotational symmetry, the eigenmodes of the experiment and computation may be rotated by an arbitrary angle around the symmetry axis. The

deviation from the exact rotational symmetry may mainly arise due to the severe influence of several ornaments at the surface of the bell. Thus, to get a realistic impression about the model validity it will, most likely, be necessary to adjust the rotation angle of numerically obtained eigenmodes with the experimental ones.
As a general conclusion it was found that the shown numerical model can be well used for the description of all relevant oscillation phenomena of the carillon bell. Future publications will comprise extensive studies about the influence of ornaments on the basis of the proposed numerical model, new bell geometries and the production of sound files out of the numerical solution. Computer simulations with the Finite Element method provide a cost-effective method to establish reliable requirements for ornaments of bells which cause a minimum of influence to the sound of the bell under consideration.

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