"Automatic" adaptation of an integration formula in thickness direction for Solid-Shell elements

Georgios Michaloudis, Karl Schweizerhof
Universität Karlsruhe, Institut für Mechanik

2007

'Automatic' adaptation of an integration formula in thickness direction for Solid-Shell elements

Georgios Michaloudis and Karl Schweizerhof

Abstract

In the simulation of shell problems in which plastic stresses in thickness direction are developed, the necessity of a more accurate integration through thickness using more integration points e.g. Gauss points becomes obvious, often increasing respectively the computational time and effort. A new algorithm is proposed for the automatic adaptation of the integration formula in thickness direction during the computation. Simple engineering criteria are found to define the time-point and the regions to increase the number of integration points.

1   Introduction

In shell problems with high stretching and intense thickness reduction general three- dimensional material laws as well as strains and stresses in thickness direction are required, imposing the use of so-called 'Solid-Shell' elements which provide the appropriate 3D-continuum discretization, see [1] and [2] . In problems such as metal forming the non-smoothness of plastic stresses imposes the use of integration rules of higher order, while mostly only globally prescribed integration rules are used. This leads, in the case of application of higher order integration rules for a complete structure, in an overintegration of elastic regions and therefore in unnecessary significant increase of the computational time. The algorithm proposed in this contribution allows the application of higher order integration formulae in an adaptive fashion reducing the error. Thus, the computation starts with the minimum required number of integration points in thickness, e.g two, and during the computation the integration points are increased only in such regions where is needed e.g. if plastic stresses occur. The method provides an efficient compromise between accuracy and computational effort.

2   Issues in adaptive integration

Except from the ordinary Gauss integration rule a compound integration rule based on Gauss rule has been implemented. This formula subdivides the interval of integration with local coordinates ζ into m subdomains as shown in Fig.1a), see [3] and [4]. The transformation of the local coordinate ζ_i of each subinterval into ζ is defined as:

$$\zeta = \frac{\zeta_i}{m} + \frac{2i+1}{m} - 1 \tag{1}$$

and the integration formula over the element thickness in the coordinate ζ takes the form of the sum:

$$\int_{V_{xyz}} f(x,y,z)\, dV = \int_{V_{\xi\eta\zeta}} f(\xi,\eta,\zeta)\, J d\xi d\eta d\zeta = \int d\xi d\eta \int f J d\zeta = \int d\xi d\eta \sum_{i=1}^{m_{sub}-1} \int_{-1}^{1} f(\xi,\eta,\zeta_i) J \frac{d\zeta_i}{m_{sub}} \tag{2}$$

As described above, the computation starts with two integration points in thickness direction, assuming a linear distribution of the stresses through thickness. A simple indicator which determines when and in which elements an increase of the number of the integration points through thickness from two to more is necessary is provided by checking the appearance of plasticity within the return mapping not with the yield criterion at the Gauss point but with its linear projection to the element's upper or lower surface, as shown in Fig.1b). At this point it must be mentioned that for the adaptation of the integration standard error estimation is not valid because this would require differentiability of the integrand up to a certain order, which is not the case for the in general not sufficiently smooth and often discontinuous function of our interest.

The increase of the number of Gauss points between computational steps necessitates the mapping of history variables from the old to the new Gauss points. The transfer of the state variables is performed with polynomial interpolation of Newton form.

Since a Gauss rule of order n yields exact result for a function of order 2n-1, the mapping from n^{old} Gauss points up to 2n new Gauss points preserves the plastic energy. This is, because the function of order 2n-1 which is defined at the 2n new Gauss points can be exactly integrated also from the n^{old} Gauss points.

The work of stresses along the element thickness, defined as $\int_{\zeta} \sigma \epsilon d\zeta = W$, has been chosen as a measure of convergence of the applied integration rule. The rate of the work is used as the criterion for the increase of the order of the integration rule which is described as:

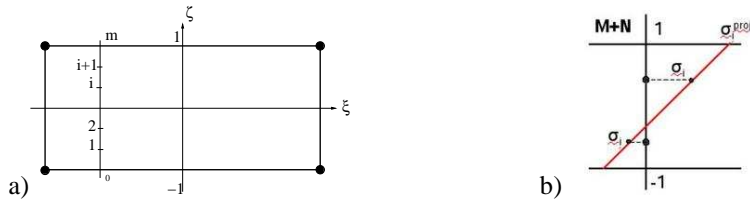$$\frac{W_k - W_{k-1}}{W_k} < tol \tag{3}$$

Figure 1: a) Compound rule based on Gauss rule b) Simple indicator based on the projection of the stress value at the Gauss points to the element's upper and lower surface.

where W_k and W_{k-1} is the work of stresses computed with the current and the previously applied integration formula, respectively. Additional convergence study has shown that in most practical cases computation with nine integration points is close to the converged result, namely $W_\infty \approx W_9$.

3 Numerical results

A simple beam under bending and normal force, as shown in Fig.2a) has been chosen to test the proposed algorithm.
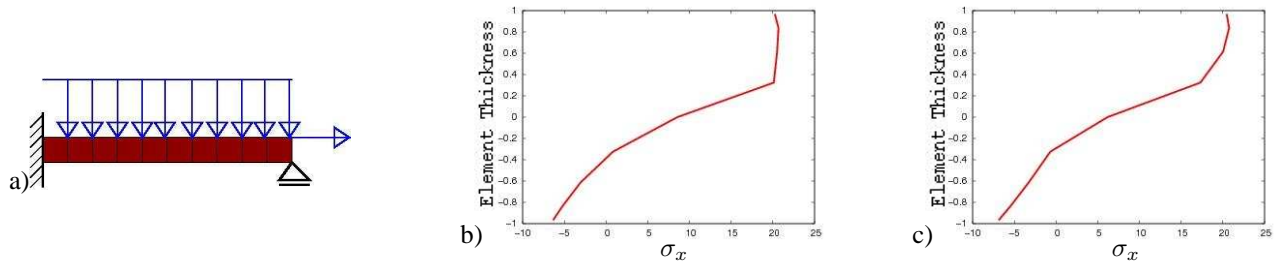

Figure 2: a) Beam under bending and normal force b) Distribution of stresses in thickness direction for the reference solution c) Distribution of stresses in thickness direction when increasing adaptively the number of Gauss points locally.

The distribution of normal stresses shown above, Fig. 2a), 2b), 2c), belongs to an element of the plastic region. The algorithm represents accurately the stress state of the element, when compared to the reference solution.

4 Conclusions

In this contribution a method for the adaptation of integration rules in thickness direction for Solid-shell elements was presented. The algorithm first detects efficiently the elements in which and the time points when an increase of the order of integration in thickness is needed. A polynomial interpolation of Newton form maps the history variables to the new Gauss points preserving the plastic energy, while the work of stresses along the thickness direction defines a criterion for the adaptation and its rate is used as a measure of convergence. Finaly, it is shown - for a simple example- that the solution converges to the reference solution and the algorithm reduces significantly the computational effort.

References

[1] R. Hauptmann, K. Schweizerhof. A systematic development of 'solid-shell' element formulations for linear and non-linear analyses employing only displacement degrees of freedom. *International Journal for Numerical Methods in Engineering*, **vol. 42**, pp. 49-69, 1998.

[2] R. Hauptmann, K. Schweizerhof, S. Doll. Extension of the 'solid-shell' concept for application to large elastic and large elastoplastic deformations. *International Journal for Numerical Methods in Engineering*, **vol. 49**, pp. 1121-1141, 2000.

[3] M. Harnau, A. Konyukhov, K. Schweizerhof. Algorithmic aspects in large deformation contact analysis using 'Solid-Shell' elements. *Computers and Structures*, **vol. 83**, pp. 1804-1823, 2005.

[4] Davis P.J., Rabinowitz P. Methods of numerical integration. *2nd ed. Academic Press;*, 1984.