Universität Karlsruhe (TH)
Forschungsuniversität · gegründet 1825

Institut für Mechanik

Adaptive Finite Element Analyses in Structural Dynamics of Shell-Like Structures – A Specific View on Practical Engineering Applications and Engineering Modeling

Karl Schweizerhof, Stephan Kizio
Universität Karlsruhe, Institut für Mechanik

2007

Institut für Mechanik
Kaiserstr. 12, Geb. 20.30
76128 Karlsruhe
Tel.: +49 (0) 721/ 608-2071
Fax: +49 (0) 721/ 608-7990
E-Mail: ifm@uni-karlsruhe.de
www.ifm.uni-karlsruhe.de

ADAPTIVE FINITE ELEMENT ANALYSES IN STRUCTURAL DYNAMICS OF SHELL-LIKE STRUCTURES – A SPECIFIC VIEW ON PRACTICAL ENGINEERING APPLICATIONS AND ENGINEERING MODELING

KARL SCHWEIZERHOF* AND STEPHAN KIZIO*

*Institut für Mechanik
Universität Karlsruhe (TH)
Englerstrasse 2, 76131 Karlsruhe, Germany
e-mail: mechanik@uni-karlsruhe.de, web page: http://www.ifm.uni-karlsruhe.de/

Key words: structural dynamics, error estimation, dual problems, adaptive meshing, engineering modeling, shell analysis

Abstract. Adaptive finite element analyses in structural dynamics require the consideration of the spatial error distribution over the complete considered time range and as well the consideration of the error due to time integration. The consideration of dual problems allows checking the error in specific quantities at certain points in time, the so-called goal-oriented error computation. On the basis of such error estimations, in principle, the adaptive modification of the finite element mesh as well as the time step is possible. However, while in a semi-discretization approach the time step could be fairly easily adjusted the modification of the finite element mesh introduces major problems. First the data have to be properly mapped between meshes avoiding non-physical artifacts and second the dual error estimation scheme has to take into account different time steps and meshes. Both actions introduce further errors into the analysis which can hardly be judged. In addition the effort for the numerical analysis concerning the computation as well as the required storage becomes overly large leading to the conclusion that adaptive analysis of real world problems based on dual error estimation cannot be handled - at least with the current computer environment.

Thus the focus of this contribution is on a discussion first on the importance of different parts of the error estimation and on the adaptive procedure and second how the major ingredients of the adaptive duality based analysis for practical engineering problems - restricting to shell problems - can still be used, regaining efficiency. For some classes of shell type problems some simplifications can be suggested while still improving the quality of the analysis considerably by adaptive procedures.

1 GOAL-ORIENTED ADAPTIVITY IN STRUCTURAL DYNAMICS

For transient loading spatially adaptive schemes for finite element computations are of high interest, as it is expected that with the evolution of an initially unknown loading a better discretization is achieved. However, some difficulties arise even for linear problems.

Within this study for the numerical solution a standard semi-discretization approach is used applying first the spatial discretization with so-called 'Solid-Shell'-elements [9] and subsequently the temporal discretization with the Newmark time integration scheme in a Galerkin type modification. Both discretization steps contribute to the total discretization error. However, as many investigations show the problems of time integration are of secondary importance as long as stability and consistency conditions are satisfied. It is also often expected that the time integration scheme affects - essentially damps out - the higher frequency contributions due to their insufficient spatial discretization.

In what follows we will thus focus on the spatial discretization error and try to construct an efficient spatial discretization with respect to an arbitrary quantity of interest. The concept of goal-oriented error computations usually consists of the derivation of identities for the error in the quantity of interest based on the solution of a dual problem for which we refer to Johnson and co-workers, see e.g. [6] and Rannacher and co-workers, see e.g. [4]. For further successful applications problems in computational mechanics see [3], [5], [7] and references therein.

The general procedure is given here for a better focus. For brevity reasons damping terms are neglected in the following. In structural dynamics the dual problem is a backward problem in time with initial conditions at the end time T which are chosen w.r.t. the quantity of interest $Q(\boldsymbol{u})$ with $\boldsymbol{u}$ as displacements. From an engineering point of view the duality approach can be interpreted as an application of the well-known dynamic reciprocity theorem in the time-domain by Graffi, see e.g. [1]. By introducing the velocities $\dot{\boldsymbol{z}}$ of the homogeneous dual backward problem - $\boldsymbol{w}$ as weighting function

$$\rho_0(\ddot{\boldsymbol{z}}, \boldsymbol{w}) + a(\boldsymbol{z}, \boldsymbol{w}) = 0 \quad \forall \boldsymbol{w} \in W \qquad \text{with } \boldsymbol{z}(T) = \boldsymbol{z}_T, \;\; \dot{\boldsymbol{z}}(T) = \dot{\boldsymbol{z}}_T \tag{1}$$

as test function for the differential equation of the spatial discretization error $\boldsymbol{e}_S$

$$\rho_0(\ddot{\boldsymbol{e}}_S, \boldsymbol{w}) + a(\boldsymbol{e}_S, \boldsymbol{w}) = \mathcal{R}_u(\boldsymbol{w}) \qquad \forall w \in W \tag{2}$$

one ends up with the identity for the error $E(\boldsymbol{u}, \boldsymbol{u}_h) = Q(\boldsymbol{u}) - Q(\boldsymbol{u}_h)$:

$$\begin{aligned} E(\boldsymbol{u}, \boldsymbol{u}_h) &= [\rho_0(\dot{\boldsymbol{e}}_S, \dot{\boldsymbol{z}}) + a(\boldsymbol{e}_S, \boldsymbol{z})]_0 + \int_0^{t_n} \mathcal{R}_u(\dot{\boldsymbol{z}}) dt \\ &= [\rho_0(\dot{\boldsymbol{z}}, \dot{\boldsymbol{e}}_S) + a(\boldsymbol{z}, \boldsymbol{e}_S)]^{t_n} \end{aligned} \tag{3}$$

The right hand side of equation 3 is now used for the proper definition of the end conditions of the dual problem at time T while the left hand side is used for the numerical error computation due to the discretization $\boldsymbol{u}_h$. Thus for time dependent problems the primal and the dual problem usually has to be solved and stored for the whole computation time $[0, T]$. In order to reduce the necessary storage memory, Fuentes et al. [8] replace the time derivatives of the primal problem by difference quotients. Thus only the displacements in the center of each time step have to be stored. Nevertheless the approach still results in a nearly insurmountable numerical effort which hampers the practical application of such methods in engineering practice. Another aspect is that due to Galerkin orthogonality the discrete dual solution $\boldsymbol{z}_h \in W^h$ is not suitable as test function, i.e. a better approximation of the dual problem is needed, see e.g. [2].

Hence for practical purposes it appears to be necessary to find simplifications of the method while retaining the important characteristic positive features of the dual problem which are needed for the adaptation of the spatial discretization. These simplifications should take into account the general physical characteristics of the considered problem. However, the application of these simplified approaches has then consequently to be restricted to particular classes of problems.

In structural dynamics we can distinguish between wave propagation and vibration problems. Wave propagation problems are characterized by the spatial and temporal transport of energy over the domain, i.e. temporal evolution terms play an important role in these problems. As a consequence for goal-oriented error estimation for wave propagation problems the dual problem has to be solved for the whole time domain $[0, T]$, since the dual weighted residual characterizes amongst others the temporal error transport. Simplifications are hardly possible and a high effort is necessary to achieve the desired goal. Thus for wave equation problems we refer to e.g. [2] and [8] and will only consider in the following simplifications for vibration type problems of shell-like structures.

2 SIMPLIFIED ERROR ESTIMATION WITH REDUCED COUPLING OF PRIMAL AND DUAL PROBLEM TOWARDS ENGINEERING APPLICATIONS

In vibration problems of shells the solution can be stated as superposition of natural modes. Hence for this class of problems it is suitable to decompose the spatial discretization error into a so-called phase error which results from the approximation of the natural frequencies and an error resulting from the approximation of the natural modes. The phase error causes the phase shift between the exact and the numerical solution which would quickly dominate the error of the time history analysis. For practical purposes when the interest is on the capability of a finite element mesh to represent a certain state of deformation and stresses correctly the phase error is of minor interest. Furthermore the phase error of the spatial discretization and the phase error of a properly chosen time integration scheme often show some cancellation effects [10]. Note that the time integration error has been neglected in the derivation of the error quantity.

Thus our approach neglects the phase error by using the right hand side of equation 3 not only for the proper definition of the end conditions of the dual problem but also for the error estimation. Practically that means that the reached state of motion is accepted and that the error estimator shall only judge the capability of the current spatial discretization to map this state w.r.t. the quantity of interest. This in turn leads to a static type dual problem that only has to be solved at the current time, see also [11]. For further simplifications the exact dual problem is replaced by the error $\boldsymbol{e}_Z = \boldsymbol{z} - \boldsymbol{z}_h$ taking advantage of the Galerkin orthogonality condition. The global error norms of the primal and the dual problem are now estimated by use of standard error estimators such as the well-known Zienkiewicz-Zhu error estimator. The numerical effort for the duality based error estimation is then similar to the effort for standard global error estimation schemes. Since only the current state of motion is considered in the error estimation the so constructed error estimator is a suitable basis for a mesh adaptation at the current time. Within the restriction to vibration type problems mesh adaptions based on the simplified error estimation approach yield efficient spatial discretizations w.r.t. the quantity of interest which is shown for various numerical examples.

REFERENCES

[1] J.D. Achenbach. *Reciprocity in Elastodynamics*. Cambridge University Press, 2003.

[2] W. Bangerth and R. Rannacher. Finite element approximation of the acoustic wave equation: Error control and mesh adaptation. *E.–W. J. Num. Math.*, 7(4):263–282, 1999.

[3] W. Bangerth and R. Rannacher. *Adaptive Finite Element Methods for Differential Equations*. Birkhäuser Verlag, 2003.

[4] R. Becker and R. Rannacher. A feed-back approach to error control in finite element methods: Basic analysis and examples. *E.-W. J. Num. Math.*, 4:237–264, 1996.

[5] R. Becker and R. Rannacher. An optimal control approach to a posteriori error estimation in finite element methods. *Acta Numerica*, 10:1–102, 2001.

[6] K. Eriksson and C. Johnson. Adaptive Finite Element Methods for Parabolic Problems I: A linear model problem. *SIAM J. of Num. Analysis*, 28(1):43–77, 1991.

[7] K. Eriksson, D. Estep, P. Hansbo and C. Johnson. Introduction to Adaptive Methods for Differential Equations. *Acta Numerica*, 4:105–158, 1995.

[8] D. Fuentes, D. Littlefield, J.T. Oden, and S. Prudhomme. Extensions of goal-oriented error estimation methods to simulations of highly-nonlinear response of shock-loaded elastomer-reinforced structures. *Comp. M. Appl. Mech. Eng.*, 195:4659–4680, 2006.

[9] R. Hauptmann, S. Doll, and K. Schweizerhof. Extension of the 'solid-shell' concept for application to large elastic and large elastoplastic deformations . *Int. J. Numer. Methods Eng.*, 49:1121–1141, 2000.

[10] T.J.R. Hughes The Finite Element Method – Linear Static and Dynamic Finite Element Analysis. Prentice–Hall International Editions 1987.

[11] S. Kizio and K. Schweizerhof. Efficient strategies for goal-oriented error estimation and mesh adaptation in structural dynamics. *Proc. CompDyn Conf.*, Greece, 2007.