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Isogeometric analysis and thermomechanical contact

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Introduction

Thermomechanical systems [1]

- Nonlinear thermoelastic material models
- Thermodynamically consistent formulation
- Energy-momentum consistent integration schemes

Thermomechanical contact and isogeometric analysis [2]

- Isogeometrical discretisation of bodies in contact
- Transient isogeometric thermomechanical contact and impact problems
- Application of Mortar concepts

First law of thermodynamics

- Lagrangian formulation of the first law

$$\dot{T} + \dot{E} = P^{\text{ext}} + Q$$

- Legendre transformation

$$E = \int_{\mathcal{B}_0} e(\boldsymbol{C}, \eta)\,\mathrm{d}V = \int_{\mathcal{B}_0} \Psi(\boldsymbol{C}, \Theta) + \Theta\eta\,\mathrm{d}V$$

- Energy balance equation

$$\underbrace{\int_{\mathcal{B}_0} \dot{\varphi}\cdot\dot{\pi}\,\mathrm{d}V}_{\dot{T}} + \underbrace{\int_{\mathcal{B}_0} \frac{1}{2}\boldsymbol{S} : \dot{\boldsymbol{C}} + \Theta\dot{\eta}\,\mathrm{d}V}_{\dot{E}} = \underbrace{\int_{\partial\mathcal{B}_0} \dot{\varphi}\cdot\boldsymbol{T}\,\mathrm{d}A}_{P^{\text{ext}}} + \underbrace{\int_{\partial\mathcal{B}_0} \boldsymbol{Q}\cdot\boldsymbol{N}\,\mathrm{d}A}_{Q}$$

- Time dependent deformation field

$$\varphi(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{I} \to \mathbb{R}^n$$

- Time dependent absolute temperature field

$$\theta(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{I} \to \mathbb{R}$$

- Virtual work of the multi-field system

$$\int_{\mathcal{B}_0} \delta\varphi\cdot\dot{\pi} + \frac{1}{2}\boldsymbol{S} : \delta\boldsymbol{C}\,\mathrm{d}V = \int_{\partial\mathcal{B}_0^T} \delta\varphi\cdot\bar{\boldsymbol{T}}\,\mathrm{d}A + \int_{\mathcal{B}_0} \delta\varphi\cdot\bar{\boldsymbol{B}}\,\mathrm{d}V$$

$$\int_{\mathcal{B}_0} \delta\theta\theta\dot{\eta} - \boldsymbol{Q}\cdot\mathrm{Grad}(\delta\theta)\,\mathrm{d}V = \int_{\partial\mathcal{B}_0^Q} \delta\theta\bar{\boldsymbol{Q}}\cdot\boldsymbol{N}\,\mathrm{d}A + \int_{\mathcal{B}_0} \delta\theta\bar{R}\,\mathrm{d}V$$

- Constitutive laws

$$\boldsymbol{\Sigma} = 2\frac{\partial\Psi}{\partial\boldsymbol{C}},\ \eta = -\frac{\partial\Psi}{\partial\theta},\ \boldsymbol{Q} = -\hat{\boldsymbol{K}}(\boldsymbol{C}, \theta)\nabla_{\boldsymbol{X}}(\theta)$$

Interface conditions

- Virtual work of the contact contributions

$$G_\varphi^c = \int_{\partial\mathcal{B}_0^{(1),c}} t_N\delta g_N + \boldsymbol{t}_T\cdot(\delta\boldsymbol{g}_T^e + \delta\boldsymbol{g}_T^s)\,\mathrm{d}A$$

$$G_\theta^c = \int_{\partial\mathcal{B}_0^{(1),c}} \delta\theta^{(1)}Q_c^{(1)} + \delta\theta^{(2)}Q_c^{(2)}\,\mathrm{d}A$$

- Normal contact

$$g_N \leq 0,\ t_N \geq 0,\ t_N g_N = 0$$

- Tangential contact

$$\hat{\phi}_c := \|\boldsymbol{t}_T\| - \mu|t_N| \leq 0,\ \dot{\zeta} \geq 0,\ \hat{\phi}_c\dot{\zeta} = 0,\ \dot{\boldsymbol{g}}_T^s = \dot{\zeta}\frac{\boldsymbol{t}_T}{\|\boldsymbol{t}_T\|}$$

- Thermal contact

$$Q_c^{(1)} = \gamma^{(1)}\boldsymbol{t}_T\cdot\dot{\boldsymbol{g}}_T^s - k_\theta|t_N|\vartheta_c,\ Q_c^{(2)} = \gamma^{(2)}\boldsymbol{t}_T\cdot\dot{\boldsymbol{g}}_T^s + k_\theta|t_N|\vartheta_c$$

- Local entropy production rate

$$\dot{\eta}_c = \frac{Q_c^{(1)}}{\theta_c^{(1)}} + \frac{Q_c^{(2)}}{\theta_c^{(2)}} \geq 0$$

Mortar method

- Lagrange multiplier field

$$\mathcal{M}^h = \{\delta\boldsymbol{t}^{(1),h} \in \mathcal{L}^2(\partial\mathcal{B}_0^{(1),c} \cap \partial\mathcal{B}_0^{(2),c})\}$$

- Discrete contact contributions

$$G_\varphi^{c,h} = \lambda_{N,A}\boldsymbol{n}\cdot\left[n^{AB}\delta\boldsymbol{q}_B^{(1)} - n^{AC}\delta\boldsymbol{q}_C^{(2)}\right] + \boldsymbol{\lambda}_{T,A}\cdot(\boldsymbol{I} - \boldsymbol{n}\otimes\boldsymbol{n})\left[n^{AB}\delta\boldsymbol{q}_B^{(1)} - n^{AC}\delta\boldsymbol{q}_C^{(2)}\right]$$

$$\begin{aligned} G_\theta^{c,h} = -\delta\Theta_A^{(1)}\Big\{&\gamma_1\boldsymbol{\lambda}_{T,B}\cdot(\boldsymbol{I} - \boldsymbol{n}\otimes\boldsymbol{n})\left[m^{ABC}\dot{\boldsymbol{q}}_C^{(1),s} - m^{ABD}\dot{\boldsymbol{q}}_D^{(2),s}\right] \\ &- k_\theta|\lambda_{N,B}|\left[m^{ABC}\Theta_C^{(1)} - m^{ABD}\Theta_D^{(2)}\right]\Big\} \\ -\delta\Theta_A^{(2)}\Big\{&\gamma_2\boldsymbol{\lambda}_{T,B}\cdot(\boldsymbol{I} - \boldsymbol{n}\otimes\boldsymbol{n})\left[\bar{m}^{ABC}\dot{\boldsymbol{q}}_C^{(1),s} - \bar{m}^{ABD}\dot{\boldsymbol{q}}_D^{(2),s}\right] \\ &+ k_\theta|\lambda_{N,B}|\left[\bar{m}^{ABC}\Theta_C^{(1)} - \bar{m}^{ABD}\Theta_D^{(2)}\right]\Big\}\end{aligned}$$

- Triple Mortar integrals

$$m^{ABC} = \int_{\partial\mathcal{B}_0^{(1),c}} R^A(\xi^{(1)})N^B(\xi^{(1)})R^C(\xi^{(1)})\,\mathrm{d}A$$

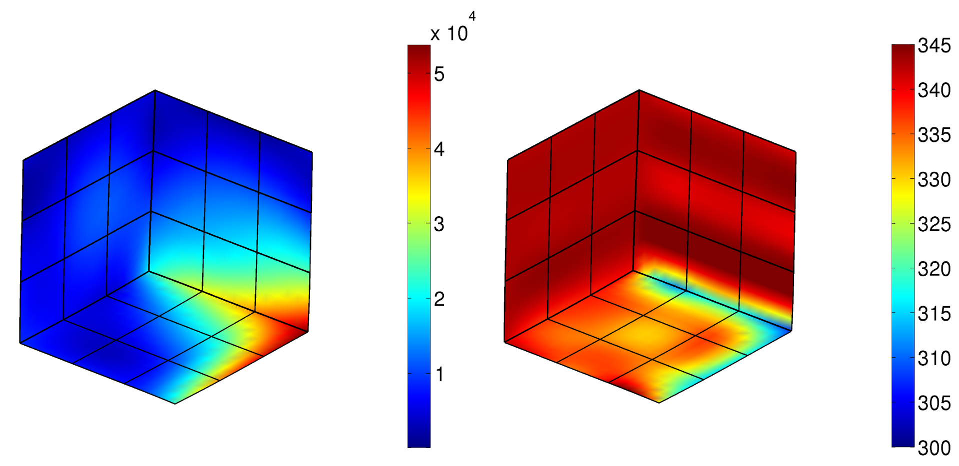
$$m^{ABD} = \int_{\partial\mathcal{B}_0^{(1),c}} R^A(\xi^{(1)})N^B(\xi^{(1)})R^D(\xi^{(2)})\,\mathrm{d}A$$

Numerical example

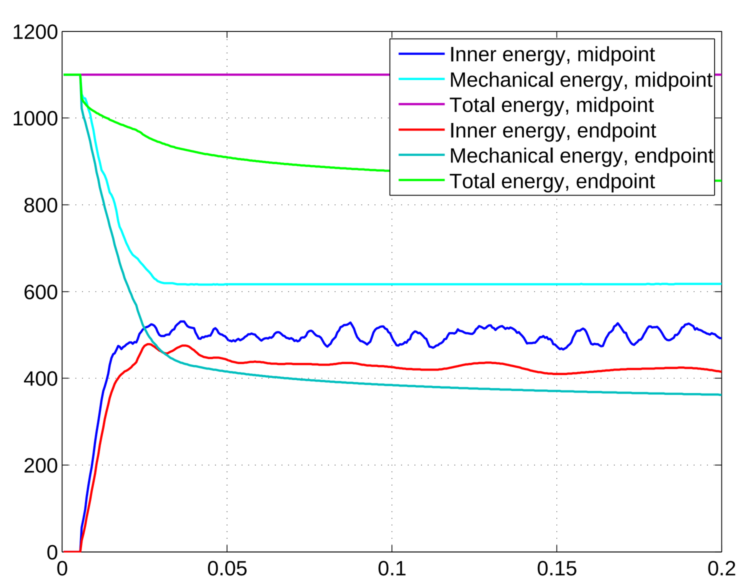
Non-linear and fully coupled Ogden material model

$$\begin{aligned}\Psi(\lambda_1, \lambda_2, \lambda_3, \theta) = &\sum_{A=1}^{3}\sum_{p=1}^{3}\frac{\mu_p(\theta_0)\frac{\theta}{\theta_0}}{\alpha_p}\left(\bar{\lambda}_A^{\alpha_p} - 1\right) + \kappa(\theta_0)\frac{\theta}{\theta_0}\beta^{-2}\left(\beta\ln(J) + J^{-\beta} - 1\right) \\ &- 3\alpha_0\,\kappa(\theta_0)\,\gamma^{-1}(J^\gamma - 1)(\theta - \theta_0) + c_0(\theta - \theta_0 - \theta\ln(\theta/\theta_0))\end{aligned}$$

Impact simulation



Von Mises stresses (left) and temperature (right) distribution.



Different energies for midpoint and endpoint rule.

References

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