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Combination of Mixed Finite Elements for Nearly Incompressible Large Deformation Solid Mechanics

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Introduction

- Element Q1P0ET2 [1]: combination of
  - Nonlinear mixed pressure element Q1P0 [3]
  - EAS formulation with transposed Wilson Modes Q1ET4 [2]

mixed pressure element Q1P0 — transposed Wilson modes

Q1P0ET2

pure displacement element Q1 — EAS formulation — two enhanced parameters

- Locking- and hourglassing-free element for nonlinear hyperelastic large deformations

Mixed Pressure Element Q1P0

- Key idea: dilatation θ as additional variable

  $\theta = J$ enforced with Lagrange Multiplier p
- Volumetric-deviatoric split of right Cauchy-Green tensor

$$\mathbf{C}^{\mathrm{STP}} = \overbrace{\theta^{2/3}}^{\text{vol}}\,\overbrace{J^{-2/3}\mathbf{C}_{\varphi}}^{\bar{\mathbf{C}}} \qquad \text{with } \sqrt{\det(\bar{\mathbf{C}})} = 1$$

- Variational potential

$$\Pi^{\mathrm{STP}}_{\mathrm{int}}(\boldsymbol{\varphi}, p, \theta) = \int_{\mathcal{B}_0} \left[\bar{W}(\bar{\mathbf{C}}) + U(\theta) + p\,(J - \theta)\right] \,\mathrm{d}V$$

- Discretization of pressure and dilatation

$$\theta^{h,e} = \text{const.} \quad \text{and} \quad p^{h,e} = \text{const.}$$

EAS Formulation Q1E4

- Key idea: enhancement of deformation gradient $\mathbf{F}$ with $\tilde{\mathbf{F}}$

$$\widehat{\mathbf{C}} = \widehat{\mathbf{F}}^{\mathrm{T}}\widehat{\mathbf{F}} \qquad \text{with } \widehat{\mathbf{F}} = \mathbf{F}_{\varphi} + \tilde{\mathbf{F}}$$

- Stress like Lagrange multiplier $\mathbf{P}$ enforces $\tilde{\mathbf{F}} = \mathbf{0}$ (eliminated in discrete formulation via orthogonality)
- Variational potential

$$\Pi^{\mathrm{EAS}}_{\mathrm{int}}(\boldsymbol{\varphi}, \mathbf{P}, \boldsymbol{\Gamma}) = \int_{\mathcal{B}_0} \left[W(\widehat{\mathbf{C}}) - \mathbf{P} : \tilde{\mathbf{F}}\right] \,\mathrm{d}V$$

- Discretization of enhanced deformation gradient with transposed Wilson modes

$$\tilde{\mathbf{F}} = \frac{j_0^{h,e}}{j^{h,e}(\xi)} \left(\mathbf{J}_0^{h,e}\right)^{-\mathrm{T}} \begin{bmatrix} \Gamma_1^{h,e}\xi & \Gamma_2^{h,e}\eta \\ \Gamma_3^{h,e}\xi & \Gamma_4^{h,e}\eta \end{bmatrix} \left(\mathbf{J}_0^{h,e}\right)^{-1}$$

Mixed Pressure EAS Element Q1P0ET2

- Definition of mixed Cauchy-Green tensor

$$\mathbf{C}^{\mathrm{A}} = \overbrace{\theta^{2/3}}^{\text{vol}}\,\overbrace{\widehat{J}^{-2/3}\widehat{\mathbf{C}}}^{\bar{\mathbf{C}}^{\mathrm{A}}} \qquad \text{with } \widehat{\mathbf{C}} = \widehat{\mathbf{F}}^{\mathrm{T}}\widehat{\mathbf{F}} \quad \text{and } \widehat{J} = \sqrt{\det(\widehat{\mathbf{C}})}$$

- Modified split neo-Hookean strain-energy function

$$W = \bar{W}(\bar{\mathbf{C}}^{\mathrm{A}}) + U(\theta) = \frac{\mu}{2}\left(\mathrm{tr}\,\bar{\mathbf{C}}^{\mathrm{A}} - 3\right) + \frac{\kappa}{2}\left(\frac{1}{2}\left(\theta^2 - 1\right) - \ln\theta\right)$$

- Variational potential with two kinematic constraints

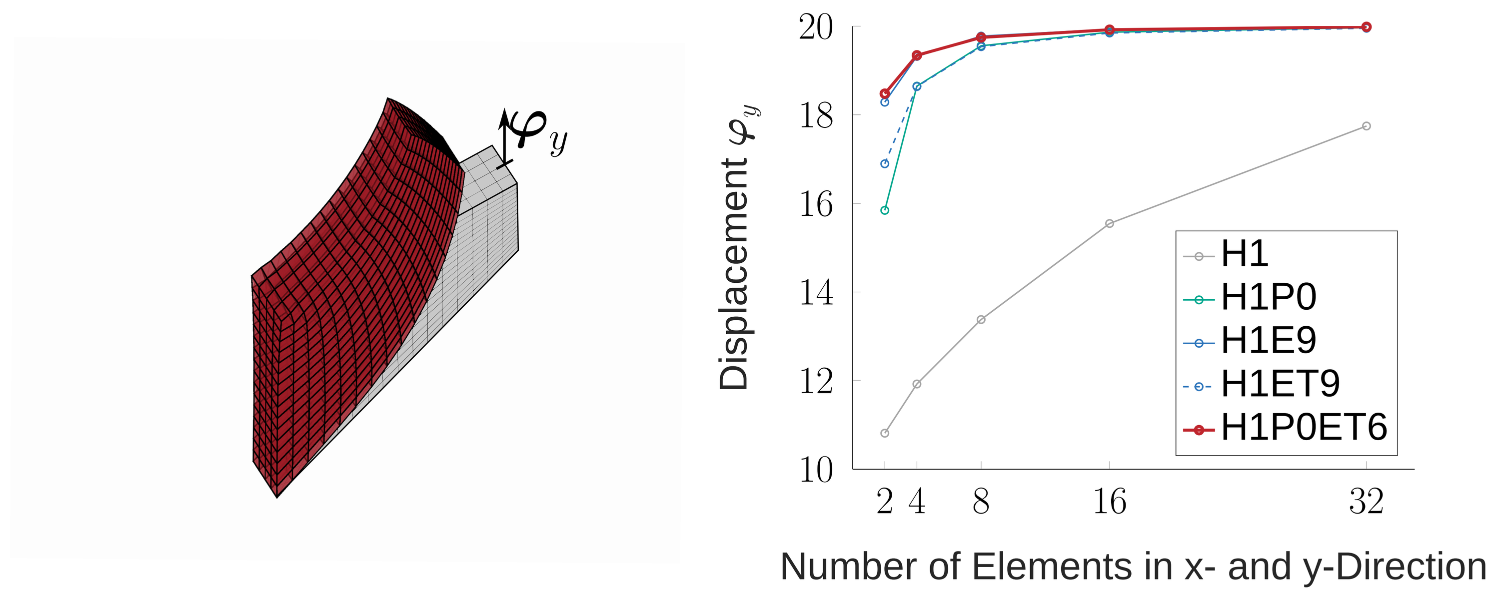
$$\Pi^{\mathrm{A}}_{\mathrm{int}}(\boldsymbol{\varphi}, \mathbf{P}, \boldsymbol{\Gamma}, p, \theta) = \int_{\mathcal{B}_0} \left[\bar{W}(\bar{\mathbf{C}}^{\mathrm{A}}) + U(\theta) - \mathbf{P} : \tilde{\mathbf{F}} + p\,(J - \theta)\right] \,\mathrm{d}V$$

- Special interpolation field for deformation gradient

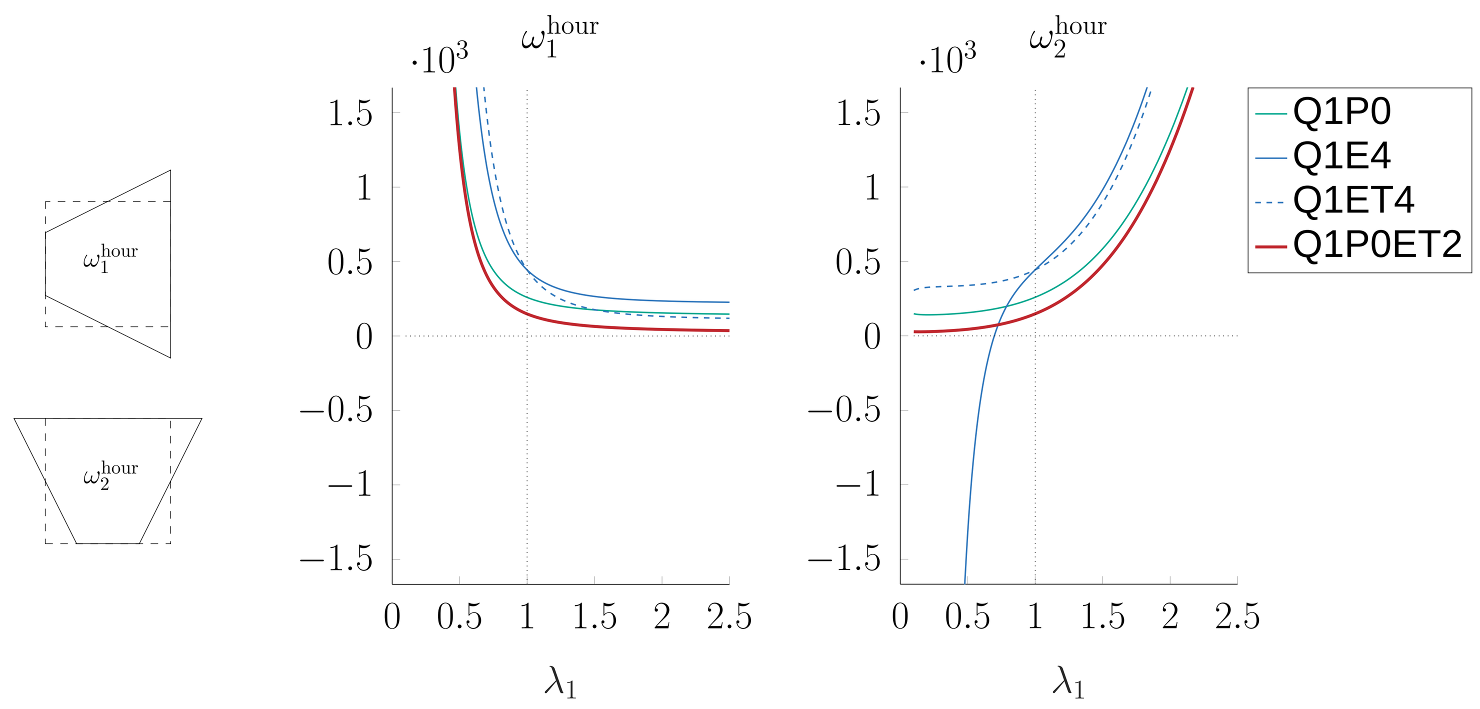
$$\tilde{\mathbf{F}} = \mathbf{F}_{\varphi,0}\frac{j_0^{h,e}}{j^{h,e}(\xi)} \left(\mathbf{J}_0^{h,e}\right)^{-\mathrm{T}} \begin{bmatrix} 0 & \Gamma_1\xi \\ \Gamma_2\eta & 0 \end{bmatrix} \left(\mathbf{J}_0^{h,e}\right)^{-1}$$

Numerical Investigations

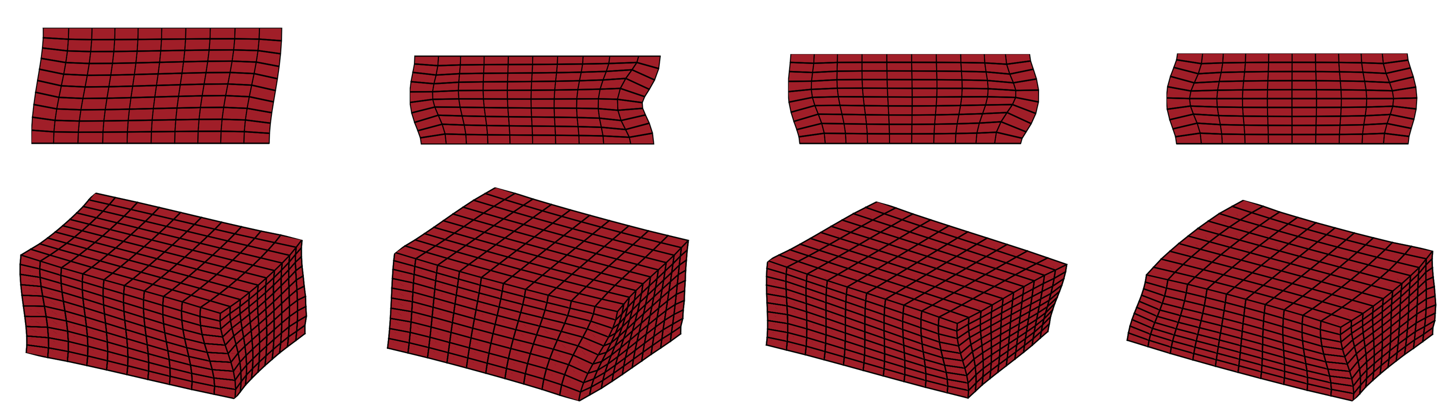
- Cooks membrane $\rightarrow$ element is locking-free



- Modal analysis $\rightarrow$ no spurious instabilities



- First four eigenmodes under compression $\rightarrow$ no hourglassing in 2D and 3D



References

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[3] Simo, J., Taylor, R., and Pister, K. Variational and Projection Methods for the Volume Constraint in Finite Deformation Elasto-Plasticity. In: *Computer Methods in Applied Mechanics and Engineering*, 51: 177–208, 1985.