Constitutive Models for Foams in Crashworthiness Analysis - a State-of-the-Art Review

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Workshop Simulation von Schaumstoffen mit stark nichtlinearem Verhalten, Hohenwart 15.-16.9.2005
Introduction

- What are foams?
  - Material scientist: any material manufactured by some expansion process
  - (Crash-) Numericist: a material with a Poisson coefficient close to zero

Both definitions coincide only for low density foams, roughly below 200g/l

- High density (>200g/l) structural foams are not foams in the numerical sense since they exhibit a non-negligible Poisson effect

- In what follows: polymeric foams are considered only
Constraints in crashworthiness analysis

- Used: explicit finite element method (LS-DYNA)
- Time step determined by element size and material tangent
- Material model determines computation time!
- Results have to be generated very fast due to development process
- Time-consuming parameter identification not acceptable
Further limitations

- **Accuracy**
  - Global density variations
  - Local density variations (gradients in the part)
  - Skin formation in cold formed parts (not in cut parts)
  - Influence of the microstructure, mainly in parts with small dimensions

- **Simulation problems**
  - Dynamic test results on soft polyurethanes (seatfoams) are dependent on size and shape of the sample, due to the open cell structure and air outflow
  - Theory of porous media needed
  - Or tests on samples that are roughly the size of the part of interest
Foams in crash simulation

Subdivision according to their mechanical behaviour

- Elastic foams
  - Hyperelastic-viscous behaviour (MAT_57/73/83...)
    \[ \sigma_i = \frac{1}{\lambda_j \lambda_k} \frac{\partial W}{\partial \lambda_i} = \frac{1}{\lambda_j \lambda_k} \tau_i(\lambda_i) \quad W = \int \tau d\lambda \begin{array}{c} + \end{array} \]

- Crushable foams
  - Visco-elastic-visco-plastic behaviour (not available yet)
  - Strain rate independent plasticity \( f(\sigma_{ij}) \leq 0 \) anisotropic (MAT_142)
  - Strain rate dependent plasticity \( f(\sigma_{vm}, p) \leq 0 \) isotropic (MAT_075)
  - Elasto-visco-plasticity \( f(\sigma_{vm}, p, \dot{\varepsilon}^p) \leq 0 \) isotropic (SAMP)
Elastic foams

- Hyperelastic viscous material behaviour
  - Poisson‘s ratio = 0 (=> principle stresses uncoupled)
  - Hill-functional + viscous terms formulated in principal stress space

\[
\sigma_i = \frac{1}{\lambda_j \lambda_k} \frac{\partial W}{\partial \lambda_i} = \frac{1}{\lambda_j \lambda_k} \tau_i(\lambda_i) \quad W = \int \tau d\lambda
\]

- Input of stress-strain curves at different strain-rates desirable

- Important applications:
  - bumper foam
  - seat and padding
  - pedestrian protection: leg impactor (Conforfoam)
Material laws for elastic foams (no Poisson effect)

- Strainrate dependent hyperelastic
- Hyperelastic-visco-elastic

**Example Equations:***
- $\sigma \propto \varepsilon$ for MAT_57, MAT_LOW_DENSITY_FOAM
- $\sigma \propto \varepsilon^2$ for MAT_83, MAT FU-CHANG_FOAM
- $\sigma \propto \varepsilon^{1.5}$ for MAT_62, MAT_LOW_DENSITY_FOAM
Material law for elastic foams (with Poisson effect)

- Implemented as MAT_SIMPLIFIED_RUBBER/FOAM in 2004 (Kolling/DuBois/Feng)
- Uses Hill instead of Ogden functional (incompressible case):

\[ W = \sum_{j=1}^{m} C_j \left[ \lambda_1^{b_j} + \lambda_2^{b_j} + \lambda_3^{b_j} - 3 + \frac{1}{n} (J^{-n b_j} - 1) \right] \]

where \( C_j \), \( b_j \) and \( n \) are material constants and \( J = \lambda_1 \lambda_2 \lambda_3 \)

The nominal stresses (force per unit undeformed area) are \( i = 1, 2, 3 \)

\[ S_i = \frac{1}{\lambda_i} \sum_{j=1}^{m} C_j \left[ \lambda_1^{b_j} - J^{-n b_j} \right] \]

for uniaxial tension:

\[ n = \frac{- \log \lambda_3}{2 \log \lambda_3 + \log \lambda_1} \quad \rightarrow \quad S_1(\lambda_1) = \frac{1}{\lambda_1} \sum_{j=1}^{m} C_j \left[ \lambda_1^{b_j} - \lambda_1^{-n b_j} \right]^{2 n + 1} \]
Material law for elastic foams (with Poisson effect)

**Foams in crash simulation**

let \[ f(\lambda) = \sum_{j=1}^{m} C_j \lambda^{b_j} \]

\[ \lambda_k S_1(\lambda_k) = f\left(\lambda_1\left(\frac{-n}{2n+1}\right)^k\right) - f\left(\lambda_1\left(\frac{-n}{2n+1}\right)^{2k}\right), \quad k = 1, 2, 3, \ldots \]

\[ f(\lambda_1) = \lambda_1 S_1(\lambda_1) + \lambda_1 \left(\frac{-n}{2n+1}\right) S_1(\lambda_1) + \lambda_1 \left(\frac{-n}{2n+1}\right)^2 S_1(\lambda_1) + \ldots \]

The function \( f(\lambda) \) is determined and

\[ S_i = \frac{1}{\lambda_i} \left[ f(\lambda_i) - f(J^{-n}) \right], \quad i = 1, 2, 3 \]

- Load curves directly inputted in material card
- Extension due to elastic damage is realized in Mat183 (for rubber first) (Kolling/Du Bois/Benson) => simulation of hysteresis by dissipation
### Material laws for elastic foams in LS-DYNA

<table>
<thead>
<tr>
<th>No.</th>
<th>keyword</th>
<th>formulation</th>
<th>input</th>
</tr>
</thead>
<tbody>
<tr>
<td>38</td>
<td>MAT_BLATZ_KO_FOAM</td>
<td>hyperel., $\nu = 0.25$</td>
<td>1 parameter</td>
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<tr>
<td>57</td>
<td>MAT_LOW_DENSITY_FOAM</td>
<td>hyperel. + viscoel.</td>
<td>LC+parameter</td>
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<tr>
<td>62</td>
<td>MAT_VISCOSOUS_FOAM</td>
<td>hyperel. + viscoel. $\nu$ variable</td>
<td>parameter</td>
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<tr>
<td>73</td>
<td>MAT_LOW_DENSITY_VISCOSOUS_FOAM</td>
<td>hyperel. + 6 viscoel. dampers</td>
<td>LC parameter</td>
</tr>
<tr>
<td>83</td>
<td>MAT_FU_CHAN_FOAM</td>
<td>hyperel.+strain-rate</td>
<td>LC/ table</td>
</tr>
<tr>
<td>177</td>
<td>MAT_HILL_FOAM</td>
<td>hyperel., $\nu$ variable</td>
<td>LC</td>
</tr>
<tr>
<td>178</td>
<td>MAT_VISCOELASTIC_HILL_FOAM</td>
<td>$= 177 +$ viscoel</td>
<td>LC + parameter</td>
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<tr>
<td>179</td>
<td>MAT_LOW_DENSITY_SYNTETIC_FOAM</td>
<td>hyperel. pseudo-damage</td>
<td>LC</td>
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<tr>
<td>180</td>
<td>MAT_LOW_DENSITY_SYNTETIC_FOAM ORTHO</td>
<td>no damage orthogonal load direction</td>
<td>LC</td>
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<tr>
<td>181</td>
<td>MAT_SIMPLIFIED_RUBBER/FOAM_(WITH_FAILURE)</td>
<td>hyperel.+strain-rate $\nu$ variable</td>
<td>LC/ table</td>
</tr>
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</table>
Material laws for elastic foams

- Material 83 is the most frequently used industrial solution for the simulation of elastic foams: bumperfoam and seatfoam
- main reason is user-friendliness: no parameters need to be fitted, test curves are (almost) directly inputted
Material laws for elastic foams

- If extrapolation of test curves is necessary:
  - We use a hyperbolic function of order n
  - Extrapolation exponent n is fitted to have a continuous transition

\[
\sigma_{n+1} = \sigma_n + \frac{\partial \sigma}{\partial \varepsilon} \left|_{\varepsilon_1} \right. \left( \frac{1-\varepsilon_1}{1-\varepsilon_n} \right)^n \Delta \varepsilon
\]

\[
\varepsilon_n > \varepsilon_1
\]

\[
n = \frac{\ln \left( \frac{\sigma_2 - \sigma_1}{\frac{\partial \sigma}{\partial \varepsilon} \left|_{\varepsilon_1} \right. \Delta \varepsilon} \right)}{\ln \left( \frac{1-\varepsilon_1}{1-\varepsilon_2} \right)}, \quad \varepsilon_2 > \varepsilon_1
\]
Example: PU-Foam

- Extremely high compression up to 98%
- Stability problems
- Time step size!
- Contact problems
- Sharp impactors cause deformation gradients in foam parts
- Lagrangean finite elements cannot follow the corresponding deformed shapes unlimitedly
- EFG methods (v970) may present an alternative
Example: Conforfoam, leg-impact

Validation test

Impact velocity 35/40 km/h

Variation in vertical positions

Tibia Acceleration

Bending Angle

Shear Displacement
Leg-impact: test configuration for validation
Foams in crash simulation

Example: Adhesive EFBond (rubber foam)

- Load cell
- Pressure plate
- Specimen
- Load direction
- Test specimen

Microstructure, 1.7x22.5mm
Foams in crash simulation

Adhesive EFBond

- $l=100\text{mm}$, $b=15\text{mm}$ and $t=2/3/4\text{mm}$
- volume elements: $\min l = 0.67\text{mm}$
- timestep $= 6.8 \times 10^{-6}\text{ s}$

Quasi static loading

Dynamic loading: $800/\text{s}$
Plastic foams

- Structural and crushable foams
- Material model: SAMP
  - not only valid for thermoplastics
  - It covers metals as well
  - Also suitable for
    - Adhesives (if you have a glue how to model)
    - Structural foams
    - Crushable foams
- Example: validation of a high-strength, low-density, expandable epoxy polymer (TeroCore by CORE Products) using a single material input card of SAMP
# Material laws for crushable foams in LS-DYNA

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<td>parameter</td>
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<td>MAT_HONEYCOMB</td>
<td>anisotropic, el-pl</td>
<td>LC</td>
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<td>53</td>
<td>MAT_CLOSED_CELL_FOAM</td>
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<td>LC</td>
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<td>MAT_CRUSHABLE_FOAM</td>
<td>isotropic, el-pl $\nu$ variable</td>
<td>LC / table</td>
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<td>MAT_BILKHU/DUBOIS_FOAM</td>
<td>isotropic, el-pl strain-rate</td>
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<td>MAT_PITZER_CRUSHABLE_FOAM</td>
<td>isotropic, el-pl $\nu$ variable</td>
<td>LC + strain-rate parameter</td>
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<td>user</td>
<td>MAT_SAMP</td>
<td>isotropic, el-pl $\nu$ variable</td>
<td>LC / table</td>
</tr>
</tbody>
</table>
Foams in crash simulation

SAMP: A Semi-Analytical Model for Polymers

In co-operation with André Haufe (Dynamore) & Paul Du Bois (Consultant)

\[
f = q^2 - A_0 - A_1 p - A_2 p^2
\]

\[
A_0 = 3\sigma_s^2, \quad A_1 = 9(\sigma_s^2 \frac{\sigma_c - \sigma_t}{\sigma_c \sigma_t})
\]

\[
A_2 = 9(\frac{\sigma_t \sigma_c - 3\sigma_s^2}{\sigma_t \sigma_c})
\]

plastic potential:

\[
g = \begin{cases} 
q^2 - A_0 - A_1 p - A_2 p^2 & \text{associated} \\
\sqrt{q^2 + \alpha p^2} & \text{non-associated}
\end{cases}
\]

flow parameter correlates to plastic Poisson’s ratio: \(\alpha \propto \nu_p = \frac{9 - 2\alpha}{18 + 2\alpha} \leq 0.5\)
SAMP – A Semi-Analytical Model for Polymers

- Hardening curves: tabulated data

- Tensile hardening curve from tensile test at different strain rates
  \[ \varepsilon_{pt} = \varepsilon_t - \frac{\sigma_t}{E}, \quad \varepsilon_t = \ln \frac{l}{l_0} \]

- Compressive hardening curve from compression test
  \[ \varepsilon_{pc} = \varepsilon_c \frac{\sigma_c}{E}, \quad \varepsilon_c = -\ln \frac{l}{l_0} \]

- Shear hardening curve from shear test
  \[ \varepsilon_{ps} = \varepsilon_s \frac{\sigma_s}{2G}, \quad \varepsilon_s = \frac{1}{2} \int \frac{\partial \dot{x}}{\partial y} dt = \frac{1}{2} \frac{d}{h_0} \]

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Foams in crashsimulation

SAMP: Ductile damage and failure

Damage for elastic unloading is defined by a load curve $\chi(\varepsilon^{pl}) = [0,1]$

$$\sigma_{eff} = \sigma_{pl} \cdot \left(1 - \chi(\varepsilon^{pl})\right)$$

Failure onset defined by the parameter $d_c$, further fading of the element defined by $\Delta \varepsilon_{rupt}^p$
Foams in crash simulation

Stress update algorithms: NEWTON-iteration

Backward Euler return mapping or general closest-point-projection or radial return

- fully implicit algorithm
- explicit algorithm

Linearization around the elastic trial state:

\[
f_{n+1}^{k-1}(\sigma_{n+1}^{trial}) = \frac{\partial f}{\partial \Delta \lambda} d (\Delta \lambda)^{k-1} = 0
\]

\[
\Delta \lambda^k = \Delta \lambda^{k-1} + d (\Delta \lambda)^{k-1} = \Delta \lambda^{k-1} - \frac{f_{n+1}^{k-1}}{\frac{\partial f}{\partial \Delta \lambda}}
\]
Materialvalidierung mit SAMP

Foams in crashsimulation

- **FOAM compression**
  - Experiment
  - Simulation

- **FOAM tension**
  - Fading out via damage formulation

- **FOAM shear**
  - Experiment
  - Simulation

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Conclusions and outlook

- Elastic foams
  - Popular material law based on strain rate dependent hyperelasticity (kind of pseudo viscosity): FU_CHANG_FOAM
  - Stress-strain curves as input directly from test data
  - Real viscosity and elastic rebound are the biggest stumbling blocks in that kind of formulation

- Crushable foams
  - Material laws for crushable foams available (even anisotropic)
  - SAMP as an alternative considering different behaviour under tension, compression, shear and biaxial loading
  - Anisotropic extension for SAMP desirable
Acknowledgement:
The experimental testing of EPP has been performed by T. Gerster, EMI
The experimental testing of EFBOND has been performed by H. Nahme, EMI
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