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ASYMPTOTIC APPROXIMATIONS FOR BUCKLING LOADS OF STIFFENED RECTANGULAR PLATES WITH VARIOUS EDGE CONDITIONS[1]

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Using perturbation techniques, the partial eigenvalue problem of orthotropic plates can be solved approximately, replacing it by a successive system of ordinary boundary value problems.

1. General Relations

Let us consider a longitudinally stiffened rectangular plate of length a and width b with constant compressive force N_x per unit width. If all longitudinal ribs are spread over the plate surface we are led to the frequently applied concept of an orthotropic plate with different rigidities B_1, B_2 and B_3 (1). The corresponding partial differential equation for the deflection w in cartesian coordinates reads

$$B_1 \frac{\partial^4 w}{\partial x^4} + 2B_3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + B_2 \frac{\partial^4 w}{\partial y^4} + N_x \frac{\partial^2 w}{\partial x^2} = 0. \tag{1}$$

Only in the case of simply supported edges the partial eigenvalue problem can be solved explicitly by separating variables. This paper presents an approach to the approximate estimation of the buckling load of an orthotropic plate with arbitrary boundary conditions under the basic assumption

$$B_1 \gg B_3,\ B_2. \tag{2}$$

This inequality is fulfilled in many practical constructions because of the large bending rigidity of the longitudinal stiffeners with respect to the small stiffnesses of the sheet itself. Without loss of generality we can choose the simplest model for the stiffness relations [1], namely

$$\begin{aligned} B_1 &= B + B_0, \\ B_2 &= B, \\ B_3 &= B, \end{aligned} \tag{3}$$

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where B is the stiffness of the sheet and B_0 results from the smeared bending rigidities of all longitudinal ribs. The parameter

$$\varepsilon = B/B_0 \ll 1 \tag{4}$$

is assumed to be a very small number, so that the inequality (2) holds. For many stiffened steel constructions the magnitude of ε has an order of about 0.01. Now we introduce dimensionless coordinates

$$\xi = \frac{\pi}{a} x, \qquad \eta = \frac{\pi}{b} y \tag{5}$$

so that the integration region is a square of length π lying in the positive quadrant. Derivatives with respect to ξ are indicated by a prime and to ξ by a dot, respectively. Independent of the actual Euler-buckling load of the stiffeners we use the second Euler-load

$$P_2 = B_0(\pi/a)^2 \tag{6}$$

to get a dimensionless load parameter

$$N = N_x/P_2. \tag{7}$$

Inserting (3) to (7) into (1) yields

$$w^{IV} + Nw^{II} = -\varepsilon\tilde{\Delta}\tilde{\Delta}w \tag{8}$$

with the modifield Laplace operator

$$\tilde{\Delta}(\,) = (\,)'' + \left(\frac{a}{b}\right)^2 (\,)^{\cdot\cdot}. \tag{9}$$

The new form (8) of the original equation (1) immediately allows the discussion of a limit case which is intuitively clear. If either the stiffness B of the sheet is extremely small or the bending rigidities B_0 of the stiffeners are extremely large, respectively, the parameter $\varepsilon = B/B_0$ tends to zero. Then (8) decomposes to an ordinary differential equation describing the buckling of the stiffeners as Euler-beams. This is called the approximation of zeroth order of the orthotropic plate problem. We are interested in higher approximations depending on a nonzero but small perturbation parameter ε. Before starting with perturbation techniques, we still have to prevent the ratio a/b in (9) from growing too large. Otherwise, the magnitude of the products $\varepsilon(a/b)^2$ and $\varepsilon(a/b)^4$ on the right side of (8), even for small ε, would be of the same order as the terms on the left side. In practice, steel constructions have ratios $0 \leq a/b < 1$, which are sufficiently small.

2. Regular perturbation analysis

We now expand in the sense of perturbation techniques [2] the deflection w and the eigenvalue N in power series in ε. Inserting the arrangement

$$w(\xi, \eta, \varepsilon) = \sum_{K=0}^{\infty} \varepsilon^K w_K(\xi, \eta), \quad N(\varepsilon) = \sum_{K=0}^{\infty} \varepsilon^K N_K \tag{10}$$

into (8) and equating equal powers in ε, we are led to an infinite system of partial differential equations

$$\begin{aligned} &\varepsilon^0: w_0^{IV} + N_0 w_0'' = 0, \\ &\varepsilon^1: w_1^{IV} + N_0 w_1'' = -(\tilde{\Delta}\tilde{\Delta}w_0 + N_1 w_0'') \\ &\varepsilon^n: w_n^{IV} + N_0 w_n'' = -\left(\tilde{\Delta}\tilde{\Delta}w_{n-1} + \sum_{K=1}^{n} N_K w_{n-K}''\right) \end{aligned} \tag{11}$$

which allow a successive solution. For this purpose we need the boundary conditions of the orthotropic plate. Expanding the original conditions corresponding to the original equation (8) in the sense of (10.1) we get for each equation of (11) the same boundary conditions as the original ones. In order to solve (11.1) only the boundary conditions at $\xi = 0$ and $\xi = \pi$ of the orthotropic plate are needed, the solution of this ordinary eigenvalue problem can easily be given and generally reads

$$\begin{aligned} w_0 &= A(\eta) f(\xi), \\ N_0 &= N_{0c}. \end{aligned} \tag{12}$$

The function $f(\xi)$ is the deflection curve of the Euler-problem of the stiffners with given supports at the ends $\xi = 0$ and $\xi = \pi$. The unknown deflection amplitude A depends on η. Because of the small number ε, we can take here and in the following N_{0c} as the lowest eigenvalue. This corresponds to the lowest buckling mode of the orthotropic plate. No difficulties would arise even in taking higher modes.

Inserting the above calculated approximation of the zeroth order in Eq. (11.2) of the first order produces an ordinary inhomogeneous boundary value problem in ξ where all functions depending an η can be considered as constants.

$$w_1^{IV} + N_{0c} w_1'' = \sum_{i=1}^{4} g_i(\eta) h_i(\xi); \quad B \cdot C|_{\xi=0;\,\xi=\pi} \tag{13}$$

The value N_{0c} and the right side of (13) are known from (12). The constants $g_i(\eta)$ depend only on A and its derivatives with respect to η. The functions $h_i(\xi)$ depend only on the known deflection curve $f(\xi)$ and its known derivatives with respect to ξ. Inserting the boundary conditions at the supports $\xi = 0$ and $\xi = \pi$ in the general solution of (13) we are led to an ordinary homogeneous differential equation for the amplitude $A(\eta)$ containing N_1 as an unknown eigenvalue. Here for the first time boundary conditions for the orthotropic plate at the edges $\eta = 0$ and $\eta = \pi$ are needed, so that this eigenvalue problem generally reads

$$A'''' + C_1 A'' + C_2(N_{0c} - N_1)A = 0; \quad B\ C|_{\eta=0;\ \eta=\pi} \tag{14}$$

The constants c_1 and c_2 are only influenced by the type of supports at $\xi = 0$ and $\xi = \pi$. The lowest eigenvalue of (14) $N_1 = N_{1c}$ is implicitly given by a transcendental equation. At this stage the first approximation of the plate buckling problem is derived, and we are interested in the quality of the approximate approach. This can easily be done in the case of a simply supported orthotropic plate where an explicit calculation of all terms in both series of (10) is possible up to arbitrarily high approximations. The buckled mode is given by

$$w = (D_0 + \varepsilon D_1 + \varepsilon^2 D_2 + \ldots) \sin\xi \sin\eta = D \sin\xi \sin\eta \tag{15}$$

with D_i, $i = 0, 1, 2, 3\ldots$ and D as integration constants. The normalized buckling load is given by

$$N = N_{0c} + \varepsilon N_{1c} = 1 + \varepsilon \left[1 + \left(\frac{a}{b}\right)^2\right]^2; \quad N_{kc} = 0, \quad k = 2, 3, 4\ldots \tag{16}$$

Evidently, the first approximations (15) and (16) is identical with the exact solution of the problem [1]. The approach converges in the case of simply supported edges very rapidly even when ε and a/b are arbitrarily large numbers. In the case of arbitrary boundary conditions it can therefore be expected that, for small ε and not too large ratios a/b, the first approximation $N = N_{0c} + \varepsilon N_{1c}$ for the buckling load is sufficiently accurate for practical purposes.

3. Results

The explicit solutions of the three boundary value problems (12), (13) and (14) are well known and N_{1c} is found numerically[3]. In the following only some final results are presented. For the sake of presentation, we divide the first order approximation $N = N_0 + \varepsilon N_1$ of (10) by N_0 to get

$$\frac{N}{N_0} = 1 + \varepsilon \frac{N_1}{N_0}. \tag{17}$$

Resubstituting (6), (7) an (4) into (17) and introducing a correction factor

$$c = N_1 / N_0 \tag{18}$$

yields the buckling formula

$$N_{1,cr} = P_{e,cr}\left(1 + c\frac{B}{B_0}\right). \tag{19}$$

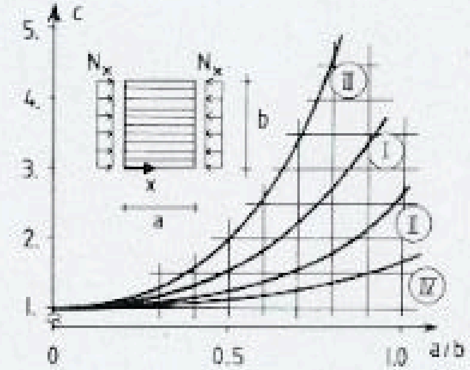

Fig. 1. Correction factor
I. all edges simply supported; $P_{e,cr} = B_0(\pi/a)^2$;
II. all edges clamped; $P_{e,cr} = 4B_0(\pi/a)^2$;
III. simply supported at $x = 0, a$; clamped at $y = 0, b$: $P_{e,cr} = B_0(\pi/a)^2$;
IV. clamped at $x = 0, a$; simply supported at $y = 0, b$: $P_{e,cr} = 4B_0(\pi/a)^2$.

Obr. 1. Korekčný koeficient
I. všetky okraje proste podopreté; $P_{e,cr} = B_0(\pi/a)^2$
II. všetky okraje votknuté: $P_{e,cr} = 4B_0(\pi/a)^2$
III. proste podopreté na okraji $x = 0, a$; votknutý na okraji $y = P_{e,cr} = B_0(\pi/a)^2$.
IV. votknutý na okraji $x = 0, a$; prosto podopretý na okraji $y = 0, b$: $P_{e,cr} = 4B_0(\pi/a)^2$.

Рис. 1. Поправочный коэффициент
I. простое опирание вдоль всех краев; $P_{e,cr} = B_0(\pi/a)^2$
II. защемление вдоль всех краев; $P_{e,cr} = 4B_0(\pi/a)^2$
III. простое опирание на крае $x = 0, a$; защемление на крае $y = 0, b$: $P_{e,cr} = B_0(\pi/a)^2$
IV. защемление на крае $x = a$; простое опирание на крае $y = 0$: $P_{e,cr} = 4B_0(\pi/a)^2$.

The value $P_{e,cr}$ is the actual Euler-buckling load of the stiffeners themselves. In fig. 1. the correction term c, depending on the ratio a/b, is given for several boundary conditions.

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