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# 'SOLID-SHELL' ELEMENTS WITH SURFACE CONTACT FORMULATION FOR LARGE DEFORMATION CONTACT PROBLEMS

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**Key words:** Higher Order 'Solid-Shell' Elements, Large Deformations, Penalty Method, Augmented Lagrangian Method, Internal Geometry of the Contact Surfaces, Contact Problems, Sheet Metal Forming

**Abstract.** *Finite elements according to the so-called 'Solid-Shell' concept as described in [2] among others e.g. [5], [4] are 3D shell elements with only displacement degrees of freedom. A special application for the 'Solid-Shell' elements are sheet metal forming problems with high stretching and large local contact pressure where standard 2D-shells fail to converge resp. give reasonable results. To describe such kind of problems good contact formulations are necessary to introduce the contact condition of the metal sheets against the rigid tools. In this contribution the penalty method and an Augmented Lagrangian approach e.g. [3], [7], [6] are used without taking friction into account as a first step in our developments. To check the penetration into the rigid tools surface contact elements are developed with four resp. nine nodes similar to the 'Solid-Shell' element types (bilinear resp. biquadratic).*

*The contact pressure is integrated over the surface element domain and the penetration check is done at the Gauss points or alternatively a nodal penetration check is done and the interpolated values at the Gauss points are used for integration [1]. For non plane, strongly curved and partial overlapping contact geometries the order of integration and the mesh refinement as well as a continuous interpolation of contact stresses are investigated concerning the fulfillment of the contact condition. For contact surface description analytical functions are used for simple surface geometries as well as contact element in-*

terpolation for arbitrary surface geometries where additionally a special procedure for the linearization of the penalty functional resp. the contact form is given.

In this context the derivation of the tangent matrix is one of the most complicated problems. Wriggers et al [8] used mathematical software which allows to obtain a matrix in closed form, while Laursen et al [3] used a linearization in the global reference frame directly. The main idea of the current approach is to describe contact kinematics as well as necessary differential operations (Lie time derivative, linearization etc.) by means of a so-called local coordinate system only. The idea is based on the a priori knowledge of the value of penetration, which is assumed to be small. This fact allows to treat the contact conditions from the surface geometry point of view, which is very similar to shell theory, where a small thickness allows to reduce the 3D geometry to the 2D surface geometry. Thus for the derivation of a tangent matrix every differential operation is treated as a covariant derivative in the local surface coordinate system. The main advantage of this is a more algorithmic and geometric structure of the tangent matrix. Every tangent matrix contains then the information about the internal geometry of the contact surface (metrics, curvature) and can be estimated in a norm within the solution process. It is rather straightforward to distinguish between the various parts of the tangent matrix in the case of linear elements. A series of examples show, that with the obtained matrices the quadratic rate of convergence within a small load step is preserved while the contact tangent matrix can be considerably simplified.

The numerical examples focus on the effect where 3D continuum approach for the shell elements appears to be beneficial in the context with contact.

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