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Abstract

Contact between solids is usually defined in terms of the impenetrability condition and additional conditions for tangential interactions, e.g. the Coulomb friction law. Therefore, contact can be considered as an interaction between boundary surfaces of contacting bodies, which encourages us to use the apparatus of differential geometry to describe all contact conditions from the surface geometry point of view. This was successfully developed within the covariant contact description, as given in Konyukhov and Schweizerhof [3], [4]. This description is developed for the solution of quasi-statical contact problems via the iterative method of Newton's type. The necessary tangent matrices are derived before the approximation process and contain explicitly all geometrical information about contacting surfaces, such as metrics and curvature tensor components. This leads to a straightforward algorithmic implementation of contact algorithms for the contact surfaces with arbitrary geometry e.g. inherited either from a finite element mesh with various order of approximation, or directly from CAD surfaces. In the current contribution, the development of contact algorithms for the shell elements with various order of approximation of the surface is illustrated.

1 Introduction

The elasto-plastic analogy together with the penalty regularization of contact conditions was found to be a robust technique for finite element implementations. Thus for 2D frictional problems Wriggers et. al. [6] applied the return mapping algorithm to obtain all necessary characteristics in each load step. The algorithm was then linearized in the global coordinate system. Laursen and Simo [5] formulated the penalty based contact conditions and the return mapping algorithm via convective surface coordinates in 3D, but the following linearization was also performed in the global coordinate system after the discretization process. In addition, a problem of symmetry for a tangent matrix in the sticking case was detected for the 3D case with an arbitrary parameterization of the surface. The symmetry was preserved only in the case of linear approximations. Another treatment of the sticking case based on a mesh tying functional was described in Wriggers [7] obtaining the correct symmetric matrix form. A consistent procedure removing the discrepancies was obtained in Konyukhov and Schweizerhof [3], [4] applying the highly developed "apparatus" of differential geometry. Hereby the contact conditions are considered in a specially defined spatial local coordinate system which corresponds to the well-known closest point procedure. All differential operations necessary for kinematics and linearization are considered as covariant derivatives. Special attention is on the consideration of the operations and the weak form on the tangent plane. The constitutive equations for the tangential tractions within the penalty regularization, or, be then called, the evolution equations, are considered in the covariant description as a parallel translation on the contact surface. Each part of the full tangent matrix, such as the normal tangent matrix, the tangent matrix in the case of sticking and the tangent matrix in the case of sliding has a geometrical structure, and, in due course, is subdivided into main, rotational and curvature parts. The core of tangent matrices does not contain the approximation matrices explicitly which allows to implement contact elements easily regardless of the order of approximation of the contact surfaces, e.g. according to the "solid-shell" finite element family [1] and [2].

2 Spatial coordinate system and contact tractions

Within the "master-slave" approach [7], a measure of normal contact interaction is defined after the closest point procedure, i.e. the penetration of a "slave" contact point **S** into a "master" surface is checked, see Fig. 1. At the projection point **C** we define a spatial local coordinate system associated to the master surface. Any spatial vector in space can be defined as

$$\mathbf{r}(\xi^1, \xi^2, \xi^3) = \boldsymbol{\rho} + \mathbf{n}\xi^3, \tag{1}$$

where $\boldsymbol{\rho}$ is a surface vector, obtained from the surface approximation. In a finite element discretization e.g. it can be written in the following form

$$\boldsymbol{\rho} = \sum_{k=1}^{M} N_k(\xi^1, \xi^2)\mathbf{x}^{(k)}, \tag{2}$$

where $N_k(\xi^1, \xi^2)$ are shape functions and $\mathbf{x}^{(k)}$ are nodal coordinates. The set of shape functions can be either of the same order as for the finite element discretization of the contact body, or it can be constructed differently as for the case of a smooth approximation of the contact surfaces. The

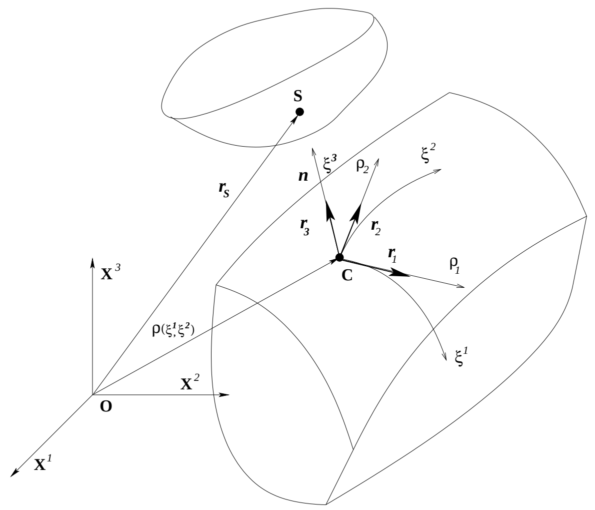

Figure 1: Introduction of the spatial coordinate system at the projection point **C**.

equilibrium equations for contact are formulated in the local coordinate system, but since contact is an interaction between surfaces then each necessary equation especially for the linearization will be considered on the tangent plane, i.e. at $\xi^3 = 0$.

2.1 Regularization of the contact conditions

The value of the penetration g as a measure of the normal contact interaction is identical to the third coordinate ξ^3. The first two convective coordinates ξ^1, ξ^2 are responsible for the tangent interaction. The regularization of the contact conditions according to e.g. Coulomb's contact law is derived in the spatial coordinate system assuming decoupling of normal and tangential tractions and it is formulated then on the tangent plane (with $\xi^3 = 0$). This leads to a non-penetration condition for the normal traction N in closed form:

$$N = \epsilon_N \xi^3, \quad iff \quad \xi^3 \leq 0, \tag{3}$$

and to the evolution equations for the tangent tractions T_i in rate form

$$\frac{dT_i}{dt} = (-\epsilon_T a_{ij} + \Gamma^k_{ij} T_k)\dot{\xi}^j - h^k_i T_k \dot{\xi}^3, \tag{4}$$

where a_{ij} and h_{ij} are components of the metric resp. curvature tensors and Γ^k_{ij} are Christoffel symbols of the master surface; ϵ_N, ϵ_T are penalty parameters for normal resp. tangential traction.

The evolution equations (4) serve to compute the trial tangent traction. The real tangent traction is recovered then by the return-mapping scheme applied for Coulomb's law.

3 Weak form and its linearization

The virtual work of contact traction is computed on the tangent plane and subdivided into a normal part and a tangential part:

$$\delta W_c = \delta W_c^N + \delta W_c^T = \int_s N \delta\xi^3 ds + \int_s T_j \delta\xi^j ds \tag{5}$$

The contact integral is computed over the "slave" surface, which is defined by a set of "slave" points and has to be linearized for a Newton type solution process according to the sticking and the sliding cases separately. The idea behind the consistent linearization is to exploit the full material time derivative in the form of the covariant derivative in the spatial coordinate system. The main points within this process are

• The convective variations are defined on the tangent plane of the spatial coordinate system via consideration of the slave point velocity as $\dot{\xi}^j = a^{ij}(\mathbf{v}_s - \mathbf{v}) \cdot \boldsymbol{\rho}_i$.

• During the linearization of $\delta\xi^i$ the derivative of the metric tensor is obtained as derivative of the spatial metric tensor considering its value on the tangent plane.

3.1 Linearization of the normal contact expression

The linearized normal part of the contact integral (5) has the following form:

$$D(\delta W_c^N) = \int_s (\delta \mathbf{r}_s - \delta\boldsymbol{\rho}) \cdot (\mathbf{n} \otimes \mathbf{n})(\mathbf{v}_s - \mathbf{v}) ds - \tag{6}$$

$$- \int_s \epsilon_N \; \xi^3 \left(\delta\boldsymbol{\rho}_{,j} \cdot a^{ij} (\mathbf{n} \otimes \boldsymbol{\rho}_i)(\mathbf{v}_s - \mathbf{v}) + (\delta \mathbf{r}_s - \delta\boldsymbol{\rho}) \cdot a^{ij} (\boldsymbol{\rho}_j \otimes \mathbf{n}) \mathbf{v}_{,i} \right) ds - \tag{6 a}$$

$$- \int_s \epsilon_N \; \xi^3 (\delta \mathbf{r}_s - \delta\boldsymbol{\rho}) \cdot h^{ij} (\boldsymbol{\rho}_i \otimes \boldsymbol{\rho}_j)(\mathbf{v}_s - \mathbf{v}) ds. \tag{6 b}$$

Here, only the matrices for the normal interaction are presented, but the structure of matrices for tangential tractions for both sticking and sliding cases remains the same, see Konyukhov and Schweizerhof [4]. The full contact tangent matrix is subdivided into the main part eqn. (6), the "rotational" part (6 a) and the "curvature" part (6 b). The last two terms are small due to the small value of the penetration ξ^3. The "rotational" part contains derivatives of $\delta\boldsymbol{\rho}$ and $\mathbf{v}$ with respect to the convective coordinates ξ^j and, therefore, represents the rotation of a master segment during the incremental solution procedure. The "curvature" part contains components of the curvature tensor h^{ij} and, therefore, represents the change of the curvature of the master surface.

4 Algorithmic implementation

The advantage of the covariant form for the tangent matrices is that only a minimal number of matrix terms are necessary for the implementation with surfaces of arbitrary order. Other parameters like metric tensor components a^{ij} and curvature tensor components h^{ij} are computed independently for the "master" contact surface. It is only necessary to define the approximation for a vector of relative displacements $\delta \mathbf{r}_s - \delta\boldsymbol{\rho}$ as well as for the derivatives of a surface vector $\boldsymbol{\rho}_i$. A "slave" node for the "node-to-surface" approach is introduced as a $(M+1)$'th node together with M nodes from the master surface approximation in eqn. (2) with N_i denoting the shape function for the node i. The following matrices are introduced

$$\mathbf{A} = \begin{bmatrix} -N_1 & 0 & 0 & -N_2 & 0 & 0 & ... & -N_M & 0 & 0 & 1 & 0 & 0 \\ 0 & -N_1 & 0 & 0 & -N_2 & 0 & ... & 0 & -N_M & 0 & 0 & 1 & 0 \\ 0 & 0 & -N_1 & 0 & 0 & -N_2 & ... & 0 & 0 & -N_M & 0 & 0 & 1 \end{bmatrix}, \qquad (7)$$

and

$$\mathbf{A}_{,j} = \begin{bmatrix} N_{1,j} & 0 & 0 & N_{2,j} & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & N_{1,j} & 0 & 0 & N_{2,j} & 0 & ... & 0 & 0 & 0 \\ 0 & 0 & N_{1,j} & 0 & 0 & N_{2,j} & ... & 0 & 0 & 0 \end{bmatrix} \qquad (8)$$

in order to define

$$\delta \mathbf{r}_s - \delta \boldsymbol{\rho} = \mathbf{A}\delta\mathbf{u} \quad \textit{resp.} \quad \mathbf{v}_s - \mathbf{v} = \mathbf{AV}, \quad \text{and} \quad \delta\boldsymbol{\rho}_{,j} = \mathbf{A}_{,j}\delta\mathbf{u} \quad \textit{resp.} \quad \mathbf{v}_{,j} = \mathbf{A}_{,j}\mathbf{V}. \qquad (9)$$

The full solution is then organized as an iterative solution of Newton's type with incremental loading.

5 Conclusion

An effective application of the covariant description for contact problems with structures defined by shell finite elements with different order of approximation is shown. Special attention is paid to the unified approach which is independent from the order of the contact surface approximation.

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