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Optimal control of multibody-systems

Introduction

Optimal control theory
- motivated by the preservation of resources & time
- leads to temporal boundary value problems

Wide range of applications
- trajectory optimization in robotics
- aerospace engineering (e.g. orbit transfers, ascent & descent trajectories)
- process engineering (e.g. chemical processes)

Our focus is set on multibody-systems (MBS), thus we deal with
- joint formulations & control input interfaces
- optimal trajectory planning for least control-effort maneuvers
- application of energy-momentum (EM) schemes

Dynamics

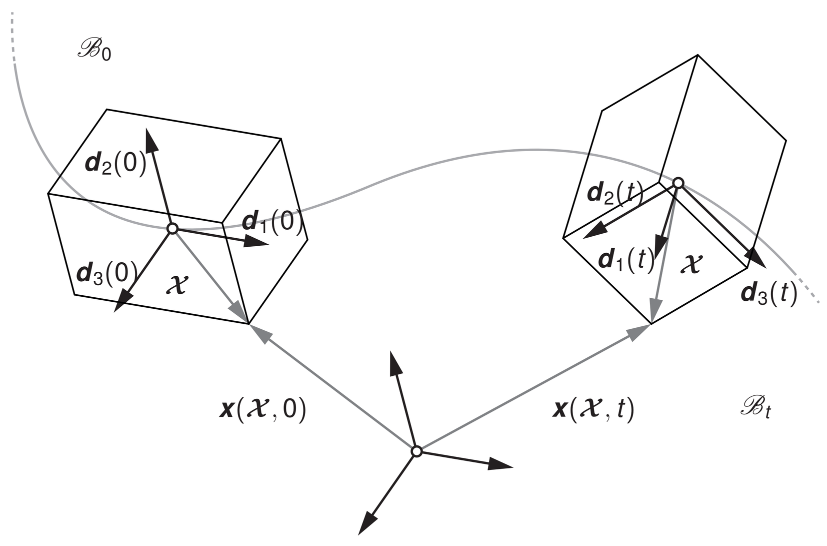
Nonsingular *direction-cosine* description of rigid body rotations

Configuration $\mathscr{B}_t$

$$\bar{\boldsymbol{x}}(\mathcal{X},t) = \boldsymbol{\varphi}(t) + \boldsymbol{R}(t)\mathcal{X}$$

with rotation matrix

$$\boldsymbol{R} \in \{\mathcal{SO}(3) | \boldsymbol{R} = \boldsymbol{d}_i \otimes \boldsymbol{e}_i\}$$



Internal constraints

$$\Phi_b^{\text{rigid}} = \boldsymbol{d}_i \cdot \boldsymbol{d}_j - \delta_{ij}$$

Discrete state equations

$$\boldsymbol{g}_n^h = \begin{bmatrix} (\boldsymbol{q}_{(n)} - \boldsymbol{q}_{(n-1)}) - \Delta t \, \boldsymbol{v}_{(n-1/2)} \\ \boldsymbol{M}(\boldsymbol{v}_{(n)} - \boldsymbol{v}_{(n-1)}) + \Delta t \left[\sum_b \nabla_{\boldsymbol{q}} \Phi_{b,(n-1/2)} \bar{\lambda}_{b,(n)} - \boldsymbol{f}(\boldsymbol{q}_{(n)}, \boldsymbol{q}_{(n-1)}, \bar{\boldsymbol{u}}_{(n)}) \right] \\ \boldsymbol{\Phi}_{(n)} \end{bmatrix}$$

This advantageous structure enables
- an object-oriented assembly framework for large MBS.
- the intuitive formulation of joints.
- the application of EM-Schemes.

Optimal Control

Discretization of the time domain $\Omega = [0, T]$

$$\Omega = \bigcup_n^N \Omega_n, \quad \Omega_n = [t_{n-1}, t_n]$$

Discrete augmented cost functional & running cost

$$J^h = \boldsymbol{\eta} \cdot \boldsymbol{\Psi}|_T + \sum_{n=1}^{N} \left[\mathcal{J}^h(\bar{\boldsymbol{u}}_n) + \boldsymbol{\mu}_n \cdot \boldsymbol{g}_n^h \right] \Delta t, \quad \mathcal{J}^h = \frac{1}{2} \bar{\boldsymbol{u}}_n \cdot \bar{\boldsymbol{u}}_n$$

Discrete necessary conditions of optimality (DNCO)

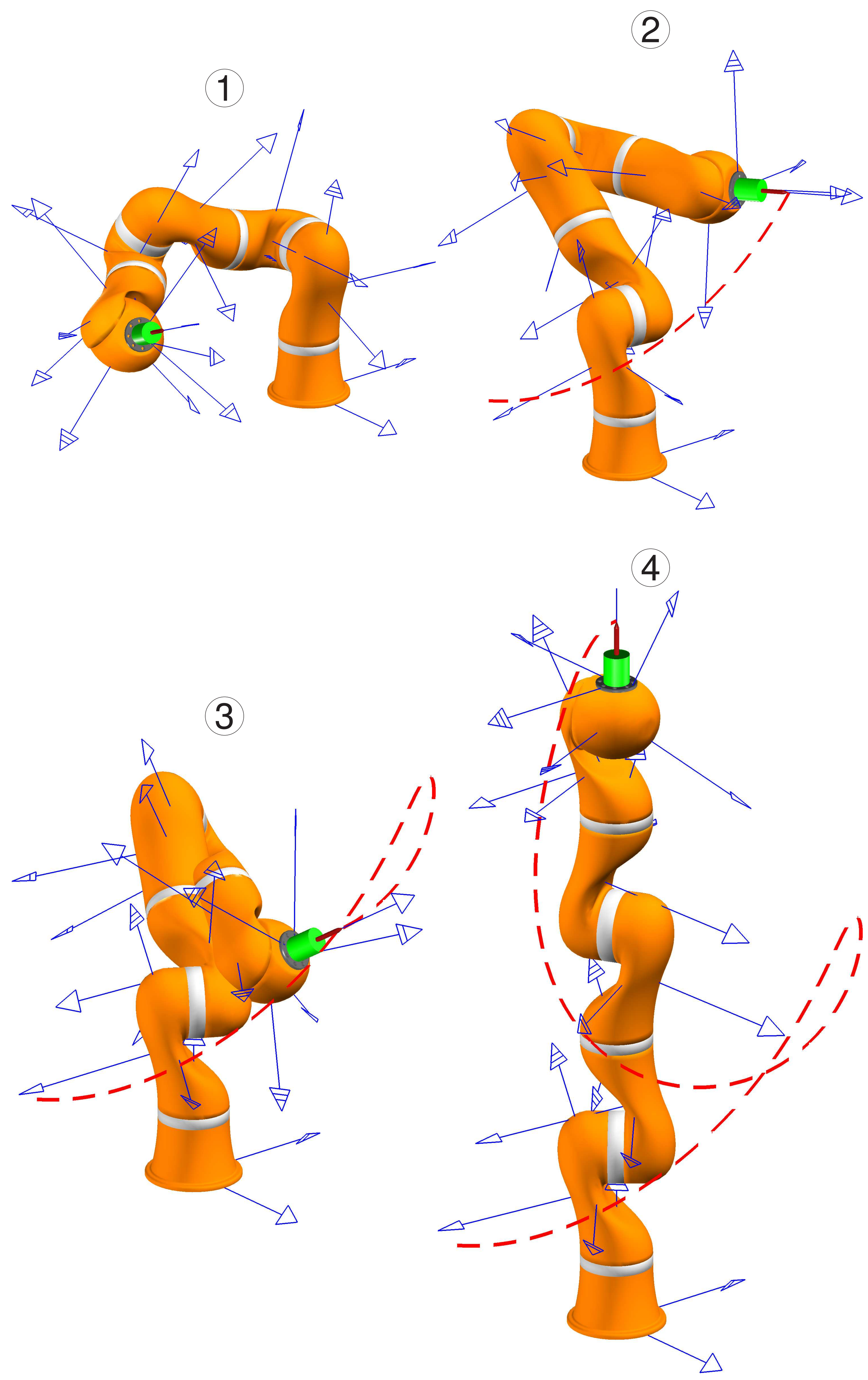
$$\delta J^h = \delta \boldsymbol{p} \cdot \left(\nabla_{\boldsymbol{p}} \tilde{\mathcal{J}}^h + (\nabla_{\boldsymbol{p}} \boldsymbol{c})^T \boldsymbol{\nu} \right) + \delta \boldsymbol{\nu} \cdot \boldsymbol{c}$$

with

$$\boldsymbol{c} = \left\{ \boldsymbol{g}_1^h, \dots, \boldsymbol{g}_n^h, \dots, \tilde{\boldsymbol{g}}_N, \boldsymbol{\Psi} \right\}$$
$$\boldsymbol{\nu} = \{ \boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_n, \dots, \tilde{\boldsymbol{\mu}}_N, \boldsymbol{\eta} \}$$
$$\boldsymbol{p} = \{ \boldsymbol{x}_1, \bar{\boldsymbol{u}}_1, \dots, \boldsymbol{x}_n, \bar{\boldsymbol{u}}_n, \dots, \tilde{\boldsymbol{x}}_N, \bar{\boldsymbol{u}}_N \}, \quad \boldsymbol{x}_n = \{ \boldsymbol{q}_n, \boldsymbol{v}_n, \bar{\boldsymbol{\lambda}}_n \}$$

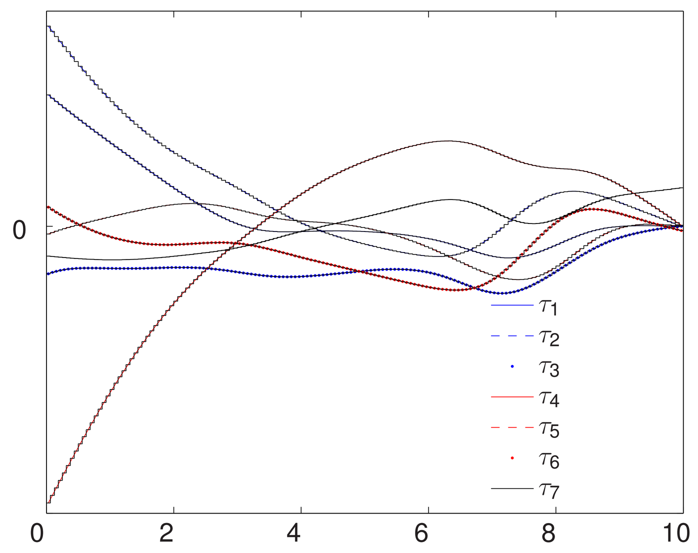
Numerical example

We consider the following MBS of a fully actuated 7-axis manipulator. The initial and terminal state ① & ④ of the MBS as well as the provided time of 10s to occupy state ④ are predefined. The time domain Ω has been subdivided into $N = 160$ equidistant subdomains. The solution of the DNCO provide the illustrated candidate solution for a least-control-effort maneuver, together with the required servo-torques to achieve this trajectory. The solution features a trajectory, where the manipulator makes perfect use of its inertia to reach the final state within the given time.

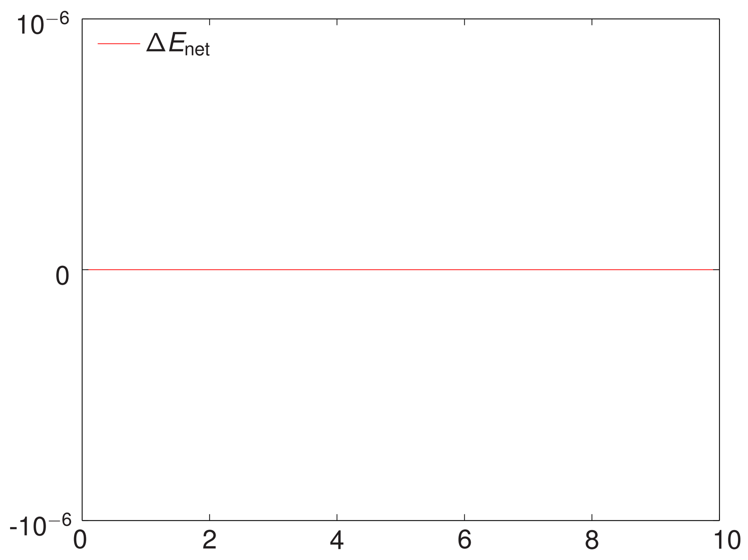


Optimal Trajectory (Least Control Effort)

The servo-torques as well as the delta net energy $\Delta E_{\text{net}}^{(n,n+1)} = E_{\text{net}}^{n+1} - E_{\text{net}}^{n}$ are shown below.



Controls



ΔE_{net} $(\epsilon_{\text{tol}} = 10^{-6})$

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