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Computational contact mechanics

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Introduction

Large deformation contact problems [1]
- Geometrical and material nonlinearities
- Inequality contact constraints
- Active set strategy to resolve the Karush-Kuhn-Tucker conditions

Consitent formulation [2]
- Mortar concept for spatial discretisation
- Complex segmentation procedure
- Energy-momentum scheme for temporal discretisation

Problem setting

- Decompsition of boundaries

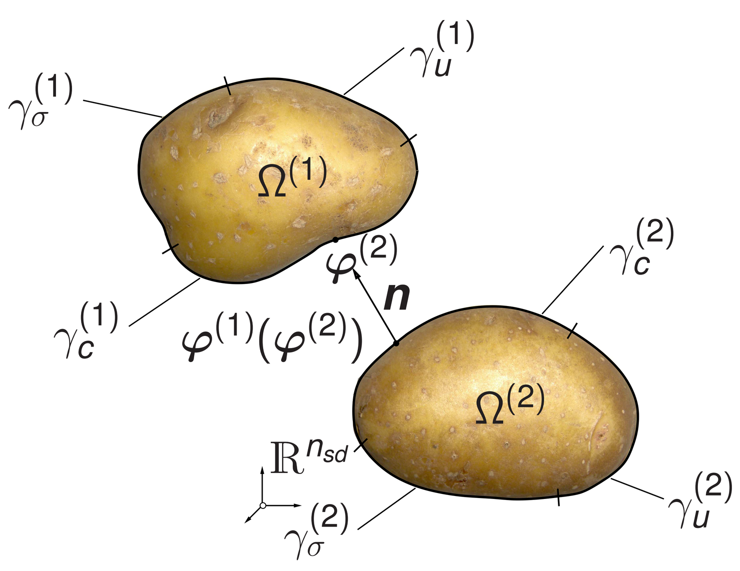
$$\gamma_\sigma^{(i)} \cup \gamma_u^{(i)} \cup \gamma_c^{(i)} = \partial\Omega^{(i)}$$

- Non-penetration condition

$$g = \boldsymbol{n} \cdot (\varphi^{(1)} - \varphi^{(2)}) \geq 0$$

- Karush-Kuhn-Tucker inequality conditions

$$t \geq 0; \quad g \geq 0; \quad tg = 0$$



- Discretization in space

$$\varphi^{\mathrm{h},(i)}(\boldsymbol{X}, t) = \sum_I N_I(\boldsymbol{X}) \boldsymbol{q}_I^{(i)}(t)$$

- Augmented Hamiltonian

$$\mathcal{H}_\lambda(\boldsymbol{p}, \boldsymbol{q}) = \frac{1}{2}\boldsymbol{p} \cdot \boldsymbol{M}^{-1}\boldsymbol{p} + V^{\mathrm{int}}(\boldsymbol{q}) + V^{\mathrm{ext}}(\boldsymbol{q}) + \boldsymbol{\lambda} \cdot \boldsymbol{\Phi}(\boldsymbol{q})$$

- Equations of motion

$$\boldsymbol{q}_{n+1} - \boldsymbol{q}_n = \frac{\Delta t}{2}\boldsymbol{M}^{-1}(\boldsymbol{p}_{n+1} + \boldsymbol{p}_n)$$
$$(\boldsymbol{p}_{n+1} - \boldsymbol{p}_n) = -\Delta t \nabla_{\mathrm{q}} V(\boldsymbol{q}_{n+\frac{1}{2}}) - \Delta t \boldsymbol{\lambda} \nabla_{\mathrm{q}} \boldsymbol{\Phi}(\boldsymbol{q}_{n+\frac{1}{2}})$$
$$\boldsymbol{0} = \boldsymbol{\Phi}(\boldsymbol{q}_{n+1})$$

Mortar method

- Weak formulation of the contact virtual work

$$G^c = \int_{\gamma_c^{(1)}} \boldsymbol{t}^{\mathrm{h},(1)} \cdot \left[\delta\varphi^{\mathrm{h},(1)} - \delta\varphi^{\mathrm{h},(2)}\right] \, \mathrm{d}\gamma$$

- Dual field

$$\boldsymbol{t}^{\mathrm{h},(1)}(\boldsymbol{X}, t) = \sum_{I \in \omega^{(1)}} N_I(\boldsymbol{X}) \lambda_I^{(i)}(t)$$

- Segment contribution of the contact constraints

$$\Phi^{\kappa}_{e_1,\mathrm{seg}}(\bar{\boldsymbol{q}}_{\mathrm{seg}}) = \boldsymbol{n} \cdot \left[\bar{n}^{\kappa\beta}\boldsymbol{q}_\beta^{(1)} - \bar{n}^{\kappa\zeta}\boldsymbol{q}_\zeta^{(2)}\right]$$

- Assembly of the segment contributions

$$\boldsymbol{\Phi}_{\mathrm{mortar}}(\boldsymbol{q}) = \mathop{\mathsf{A}}_{e_1 \in \bar{\epsilon}^{(1)}} \bigcup_{\mathrm{seg}} \boldsymbol{\Phi}_{e_1,\mathrm{seg}}(\bar{\boldsymbol{q}}_{\mathrm{seg}}) = \mathop{\mathsf{A}}_{e_1 \in \bar{\epsilon}^{(1)}} \bigcup_{\mathrm{seg}} \begin{bmatrix} \Phi^{\kappa=1}_{e_1,\mathrm{seg}}(\bar{\boldsymbol{q}}_{\mathrm{seg}}) \\ \vdots \\ \Phi^{\kappa=4}_{e_1,\mathrm{seg}}(\bar{\boldsymbol{q}}_{\mathrm{seg}}) \end{bmatrix}$$

Concept of a discrete gradient

- Reparametrization of the augmented Hamiltonian using at most quadratic invariants

$$\mathcal{H}_\lambda(\boldsymbol{p}, \boldsymbol{q}) = \widetilde{\mathcal{H}}_\lambda(\pi(\boldsymbol{p}, \boldsymbol{q}))$$

- Application of the chain rule

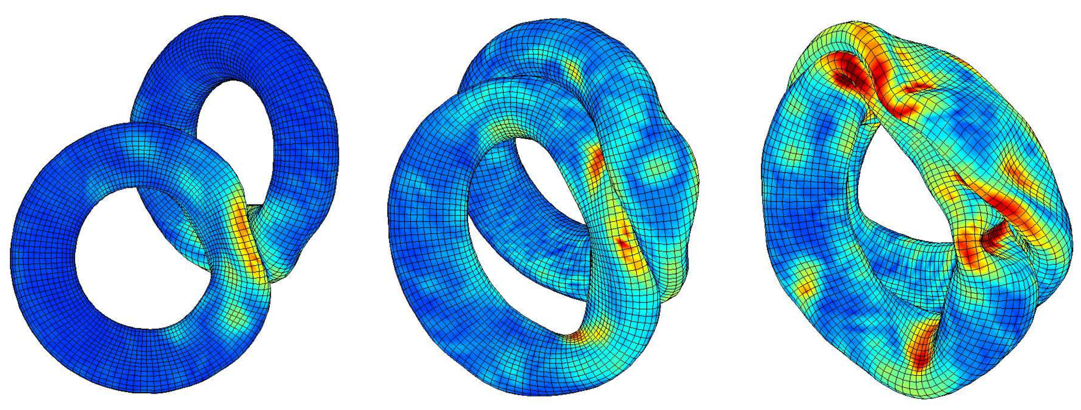
$$\overline{\nabla}_{\mathrm{q}}\mathcal{H}_\lambda = \nabla_{\mathrm{q}}\pi(\boldsymbol{p}_{n+\frac{1}{2}}, \boldsymbol{q}_{n+\frac{1}{2}})^T \overline{\overline{\nabla}}_\pi \widetilde{\mathcal{H}}_\lambda(\pi(\boldsymbol{z}_n), \pi(\boldsymbol{z}_{n+1}))$$

- G-equivariant discrete gradient

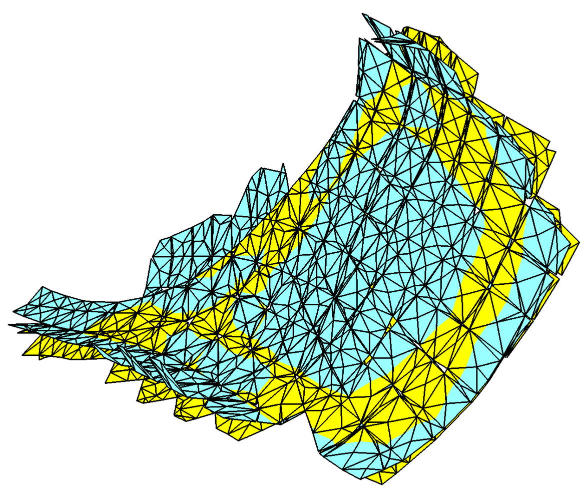
$$\overline{\overline{\nabla}}_\pi \widetilde{\mathcal{H}}_\lambda(\boldsymbol{\pi}_n, \boldsymbol{\pi}_{n+1}) = \nabla_\pi \widetilde{\mathcal{H}}_\lambda(\boldsymbol{\pi}_{n+\frac{1}{2}}) + \frac{\widetilde{\mathcal{H}}_\lambda(\boldsymbol{\pi}_{n+1}) - \widetilde{\mathcal{H}}_\lambda(\boldsymbol{\pi}_n) - \nabla_\pi \widetilde{\mathcal{H}}_\lambda(\boldsymbol{\pi}_{n+\frac{1}{2}}) \cdot (\boldsymbol{\pi}_{n+1} - \boldsymbol{\pi}_n)}{\|(\boldsymbol{\pi}_{n+1} - \boldsymbol{\pi}_n)\|^2}(\boldsymbol{\pi}_{n+1} - \boldsymbol{\pi}_n)$$

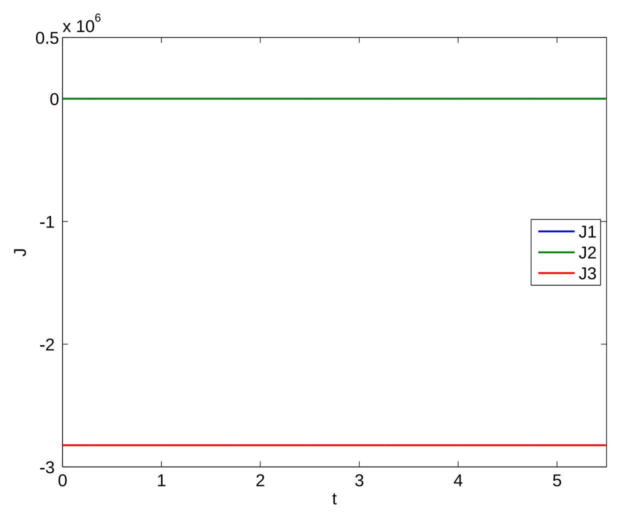
Numerical example

Impact problem of two hollow tori. The inner and the outer radius of the tori is 52 and 100, respectively. The wall thickness of each hollow torus is 4.5. Both tori are subdivided into 3120 elements, using a Neo-Hookean hyperelastic material with $E = 2250$ and $\nu = 0.3$. The initial densities are $\rho = 0.1$ and the homogeneous, initial velocity of the left torus is given by $v = [30, 0, 23]$.



Several configurations and the stress distributions are displayed. The segmentation after 2s as well as the three components of the angular momentum are shown below.





References

- P. Betsch and C. Hesch. Energy-momentum conserving schemes for frictionless dynamic contact problems. Part I: NTS method. *IUTAM Bookseries*, 3:77–96. Springer-Verlag, 2007.
- C. Hesch and P. Betsch. A mortar method for energy-momentum conserving schemes in frictionless dynamic contact problems. *Int. J. Numer. Meth. Engng*, 77:1468–1500, 2009.