Hierarchical refinement for higher-order phase-field models

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Introduction

Higher-order phase-field models[1]
- C^1 continuity of the shape functions required
- Diffuse interface representation of sharp interfaces

Multivariate NURBS shape function
- Predefined basis functions continuity
- Tensor product structure $\leftrightarrow$ local refinement

Hierarchical refinement
- Subdivision procedure to replace splines
- Maintains all global properties, i.e. smoothness/continuity

Computational modeling

- Isogeometric Analysis (IGA)

$$\varphi^h = \sum_{A\in\omega} R^A \boldsymbol{q}_A \quad \text{and} \quad c^h = \sum_{A\in\omega} R^A c_A$$

- Recursive B-Spline definition

$$N_{\mathrm{i},p} = \frac{\xi - \xi_\mathrm{i}}{\xi_{\mathrm{i}+p} - \xi_\mathrm{i}} N_{\mathrm{i},p-1}(\xi) + \frac{\xi_{\mathrm{i}+p+1} - \xi}{\xi_{\mathrm{i}+p+1} - \xi_{\mathrm{i}+1}} N_{\mathrm{i}+1,p-1}(\xi)$$

where

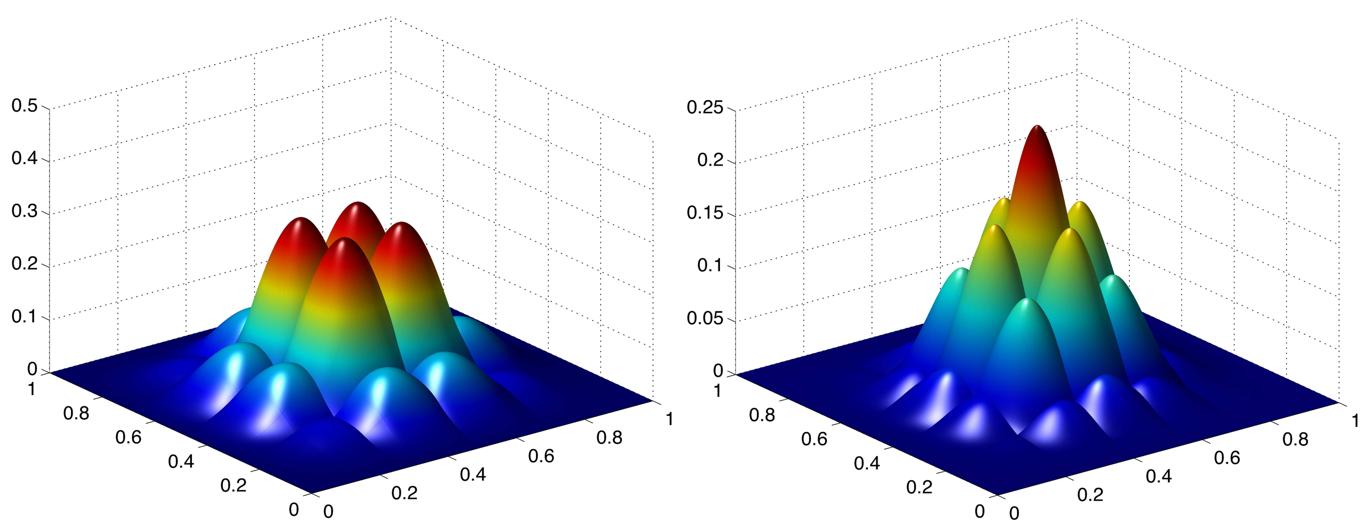
$$N_{\mathrm{i},0}(\xi) = \begin{cases} 1 & \text{if } \xi_\mathrm{i} \leq \xi < \xi_{\mathrm{i}+1} \\ 0 & \text{otherwise} \end{cases}$$

- Multivariate NURBS shape functions

$$R^A = \frac{N_{\mathrm{i},p}(\xi) M_{\mathrm{j},q}(\eta) L_{\mathfrak{k},r}(\zeta) w_{\mathrm{i,j},\mathfrak{k}}}{\sum_{\hat{\mathrm{i}}=1}^{n} \sum_{\hat{\mathrm{j}}=1}^{m} \sum_{\hat{\mathfrak{k}}=1}^{l} N_{\hat{\mathrm{i}},p}(\xi) M_{\hat{\mathrm{j}},q}(\eta) L_{\hat{\mathfrak{k}},r}(\zeta) w_{\hat{\mathrm{i}},\hat{\mathrm{j}},\hat{\mathfrak{k}}}}$$

Hierarchical refinement

- Subdivision of 2D quadratic and cubic B-Splines[2]



- B-Splines

$$B^{k,A} = B_{\boldsymbol{p}}^{k,\boldsymbol{i}}(\xi) = \sum_{\boldsymbol{j}=\boldsymbol{0}}^{\boldsymbol{p}+\boldsymbol{1}} \prod_{l=1}^{d} 2^{-p_l} \binom{p_l+1}{j_l} N_{i_l,p_l}(2\xi^l - j_l)$$

- NURBS

$$R^{k,A} = R_{\boldsymbol{p}}^{k,\boldsymbol{i}}(\xi) = \frac{\sum_{\boldsymbol{j}=\boldsymbol{0}}^{\boldsymbol{p}+\boldsymbol{1}} S_{\boldsymbol{i},\boldsymbol{j}} B_{\boldsymbol{p}}^{k+1,2\boldsymbol{i}-\boldsymbol{1}+\boldsymbol{j}}(\xi) w_{2\boldsymbol{i}-\boldsymbol{1}+\boldsymbol{j}}}{\sum_{\boldsymbol{i}} \sum_{\boldsymbol{j}=\boldsymbol{0}}^{\boldsymbol{p}+\boldsymbol{1}} S_{\boldsymbol{i},\boldsymbol{j}} B_{\boldsymbol{p}}^{k+1,2\boldsymbol{i}-\boldsymbol{1}+\boldsymbol{j}}(\xi) w_{2\boldsymbol{i}-\boldsymbol{1}+\boldsymbol{j}}}$$

- Geometric parametrization

$$\varphi^h := \mathfrak{F}(\xi) = R^A(\xi)\boldsymbol{q}_A$$

- Subdivision for B-Splines

$$\boldsymbol{B}^k \boldsymbol{q}_k = (\boldsymbol{S}\boldsymbol{B}^{k+1})\boldsymbol{q}_k = \boldsymbol{B}^{k+1} \cdot (\boldsymbol{S}^T \boldsymbol{q}_k)$$

Thermal diffusion in binary blends

- Weak form of the coupled system

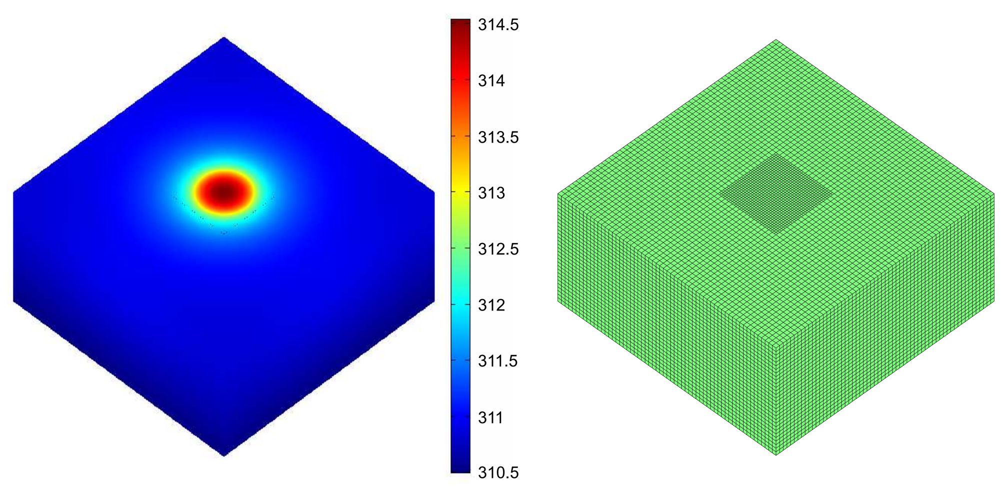
$$\rho \int_{\mathcal{B}} c_p \frac{\partial T}{\partial t} \delta T \,\mathrm{d}V + \int_{\mathcal{B}} k \nabla T \cdot \nabla \delta T \,\mathrm{d}V - \alpha \int_{\mathcal{B}} I \delta T \,\mathrm{d}V = 0$$

$$\int_{\mathcal{B}} \frac{\partial c}{\partial t} \delta c \,\mathrm{d}V + M \int_{\mathcal{B}} \nabla \partial_c \Psi^{\mathrm{con}} \cdot \nabla \delta c \,\mathrm{d}V +$$

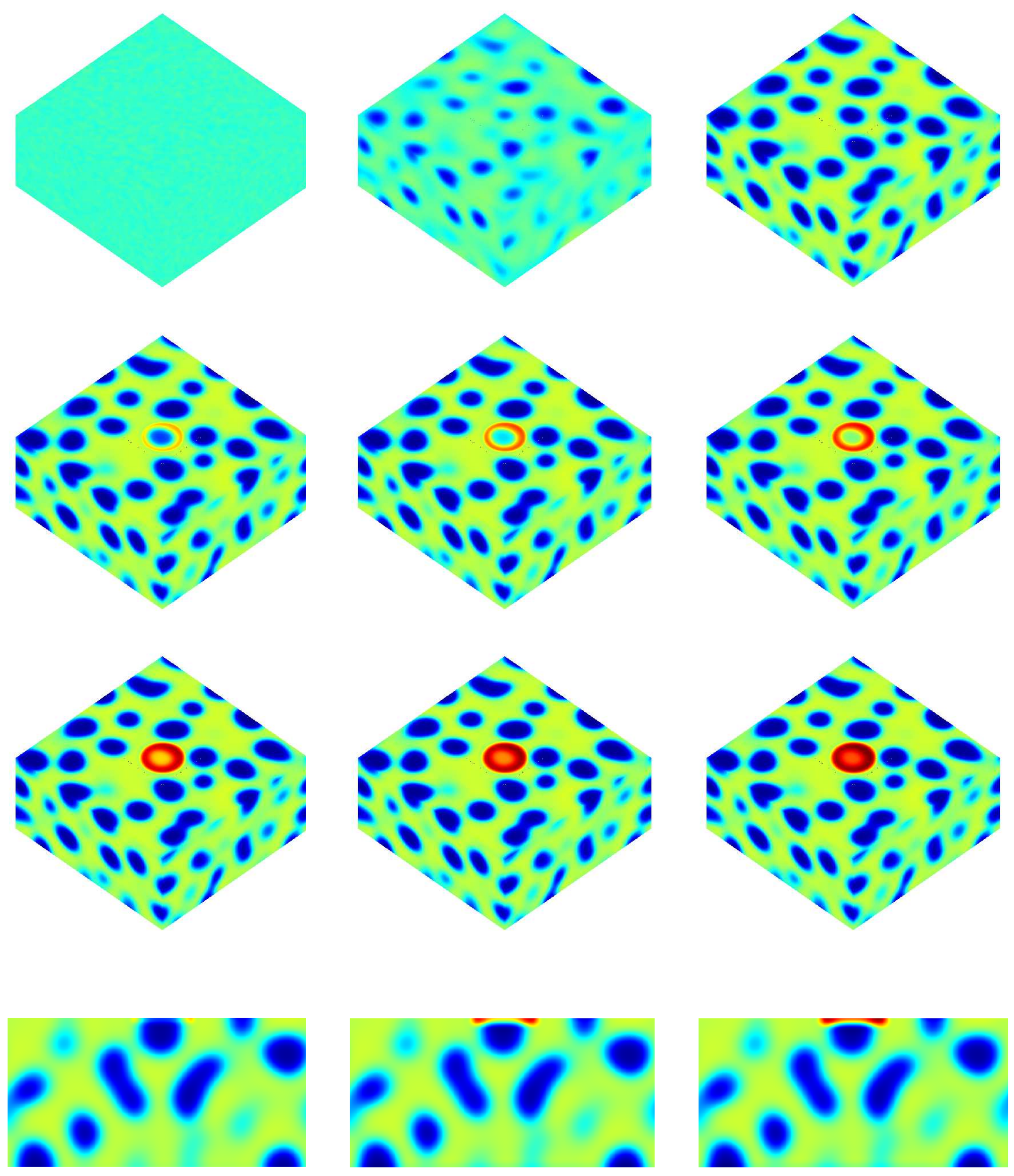
$$D_T \int_{\mathcal{B}} c(1-c) \nabla T \cdot \nabla \delta c \,\mathrm{d}V \quad + \lambda M \int_{\mathcal{B}} \Delta c \Delta \delta c \,\mathrm{d}V = 0,$$

Numerical example

- Focused laser spot on PDMS/PEMS polymer blend / local mesh refinement



- Aging-simulation without heat supply / with concentrated laser spot / profile of the heated region



References

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