Isogeometric analysis and domain decomposition

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Introduction

Multivariate NURBS shape function [1]
- Enhanced control of continuity requirements
- Link between CAD and CAE

Computational analysis [2]
- Application to finite element analysis
- Local refinement via T-Splines or subdivision methods

Domain decomposition [3]
- Mortar based implementation
- Combined use of Lagrangian and NURBS shape functions

NURBS

- Isogeometric Analysis (IGA)

$$\varphi^h = \sum_{A\in\omega} R^A \boldsymbol{q}_A, \ \delta\varphi^h = \sum_{A\in\omega} R^A \delta\boldsymbol{q}_A$$

- Multivariate NURBS shape functions

$$R^A = \frac{N_{\mathrm{i},p}(\xi) M_{\mathrm{j},q}(\eta) L_{\mathfrak{k},r}(\zeta) w_{\hat{\mathrm{i}},\hat{\mathrm{j}},\hat{\mathfrak{k}}}}{\sum_{\hat{\mathrm{i}}=1}^{n}\sum_{\hat{\mathrm{j}}=1}^{m}\sum_{\hat{\mathfrak{k}}=1}^{l} N_{\hat{\mathrm{i}},p}(\xi) M_{\hat{\mathrm{j}},q}(\eta) L_{\hat{\mathfrak{k}},r}(\zeta) w_{\hat{\mathrm{i}},\hat{\mathrm{j}},\hat{\mathfrak{k}}}}$$

- Cox-de Boor relation

$$N_{\mathrm{i},p} = \frac{\xi - \xi_{\mathrm{i}}}{\xi_{\mathrm{i}+p} - \xi_{\mathrm{i}}} N_{\mathrm{i},p-1}(\xi) + \frac{\xi_{\mathrm{i}+p+1} - \xi}{\xi_{\mathrm{i}+p+1} - \xi_{\mathrm{i}+1}} N_{\mathrm{i}+1,p-1}(\xi)$$

where

$$N_{\mathrm{i},0}(\xi) = \begin{cases} 1 & \text{if } \xi_{\mathrm{i}} \le \xi < \xi_{\mathrm{i}+1} \\ 0 & \text{otherwise} \end{cases}$$

- Subdivision of parameter space in finite elements

$$\Xi = \{\xi_1, \xi_2, \dots, \xi_{n+p+1}\}$$
$$\mathcal{H} = \{\eta_1, \eta_2, \dots, \eta_{n+p+1}\}$$
$$\mathcal{I} = \{\zeta_1, \zeta_2, \dots, \zeta_{n+p+1}\}$$

Computational mechanics

- Virtual work

$$G^{\mathrm{h}}(\boldsymbol{q}, \delta\boldsymbol{q}) = \sum_i \delta\boldsymbol{q}_A^{(i)} \cdot \left[\boldsymbol{M}^{AB}\ddot{\boldsymbol{q}}_B^{(i)} + \boldsymbol{f}^{(i),\mathrm{int},A} + \boldsymbol{f}^{(i),\mathrm{ext},A}\right]$$

- Mass matrix

$$\boldsymbol{M}^{AB} = \int_{\mathcal{B}_0^{(i)}} \rho_0 R^A R^B \,\mathrm{d}V$$

- Internal forces

$$\boldsymbol{f}^{(i),\mathrm{int},A} = \int_{\mathcal{B}_0^{(i)}} \nabla R^A \cdot \boldsymbol{S}\,\nabla R^B \,\mathrm{d}V \boldsymbol{q}_B^{(i)}, \ \boldsymbol{S} = 2\frac{\partial W(\boldsymbol{C}^{(i),\mathrm{h}})}{\partial \boldsymbol{C}^{(i),\mathrm{h}}}$$

- External forces

$$\boldsymbol{f}^{(i),\mathrm{ext},A} = -\int_{\mathcal{B}_0^{(i)}} R^A \bar{\boldsymbol{B}}^{(i)} \,\mathrm{d}V - \int_{\partial\mathcal{B}_0^{(i),\sigma}} R^A \bar{\boldsymbol{T}}^{(i)} \,\mathrm{d}A$$

- Strain energy function

$$V^{(i),\mathrm{int}}(\boldsymbol{q}^{(i)}) = \int_{\mathcal{B}_0^{(i)}} W(\boldsymbol{C}^{(i),\mathrm{h}}) \,\mathrm{d}V$$

Domain decomposition

- Balance of linear momentum across the interface

$$\int_{\partial\mathcal{B}_0^{(1),d}} \boldsymbol{t}^{(1),\mathrm{h}} \cdot (\delta\boldsymbol{\varphi}^{(1),\mathrm{h}} - \delta\boldsymbol{\varphi}^{(2),\mathrm{h}}) \,\mathrm{d}A = 0$$

- Lagrangian shape functions for the dual field

$$\boldsymbol{t}^{(1),\mathrm{h}} = \sum_{A\in\tilde{\omega}^{(1)}} N^A \boldsymbol{\lambda}_A$$

- Segment contributions of the discrete mortar constraints

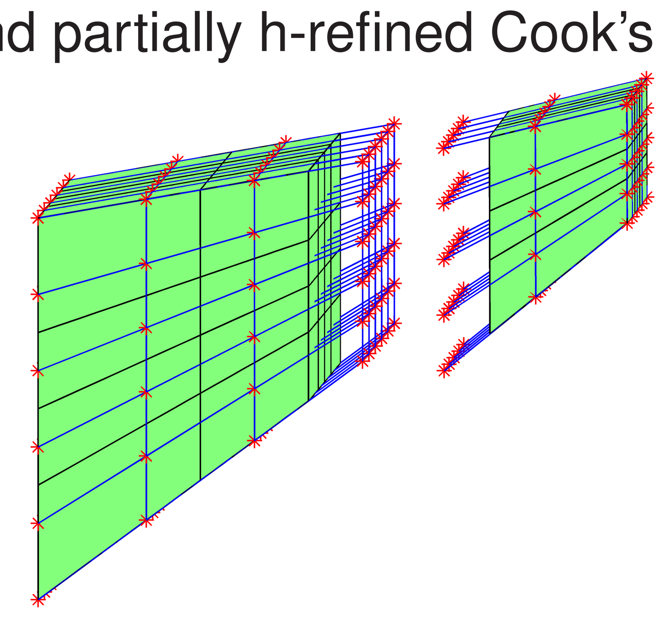
$$\boldsymbol{\Phi}_{e,\mathrm{seg}}^{\kappa} = n^{\kappa\beta}\boldsymbol{q}_\beta^{(1)} - n^{\kappa\zeta}\boldsymbol{q}_\zeta^{(2)}$$

- Mortar integrals

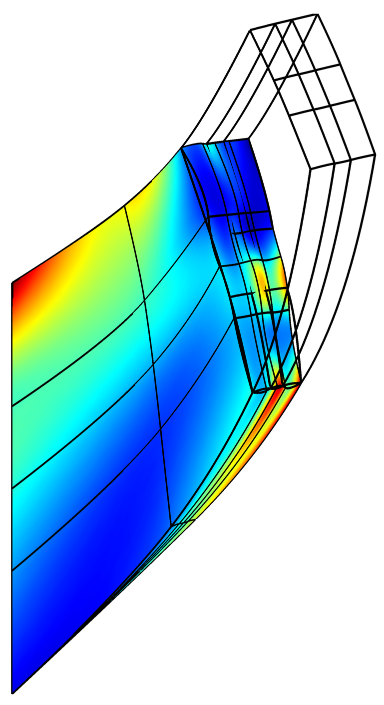
$$n^{\kappa\beta} = \langle N^\kappa(\boldsymbol{\xi}_{\mathrm{seg}}^{(1),\mathrm{h}}(\boldsymbol{\eta})) R^\beta(\boldsymbol{\xi}_{\mathrm{seg}}^{(1),\mathrm{h}}(\boldsymbol{\eta}))\rangle_{\partial\mathcal{B}_{0,\mathrm{seg}}^{(1),d}}$$
$$n^{\kappa\zeta} = \langle N^\kappa(\boldsymbol{\xi}_{\mathrm{seg}}^{(1),\mathrm{h}}(\boldsymbol{\eta})) R^\zeta(\boldsymbol{\xi}_{\mathrm{seg}}^{(2),\mathrm{h}}(\boldsymbol{\eta}))\rangle_{\partial\mathcal{B}_{0,\mathrm{seg}}^{(1),d}}$$

Numerical example

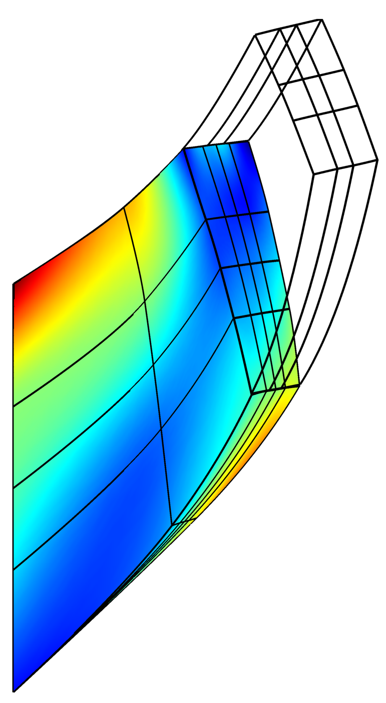
Decomposed and partially h-refined Cook's membrane



A Dirichlet boundary condition has been applied to the left surface, whereas a Neumann boundary has been applied to the right surface. Von Mises stress distribution is displayed below.



KTS method.



Mortar method.

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