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# Finite Element Analysis of a Bar Element Made of Linear Thermoviscoelastic Material

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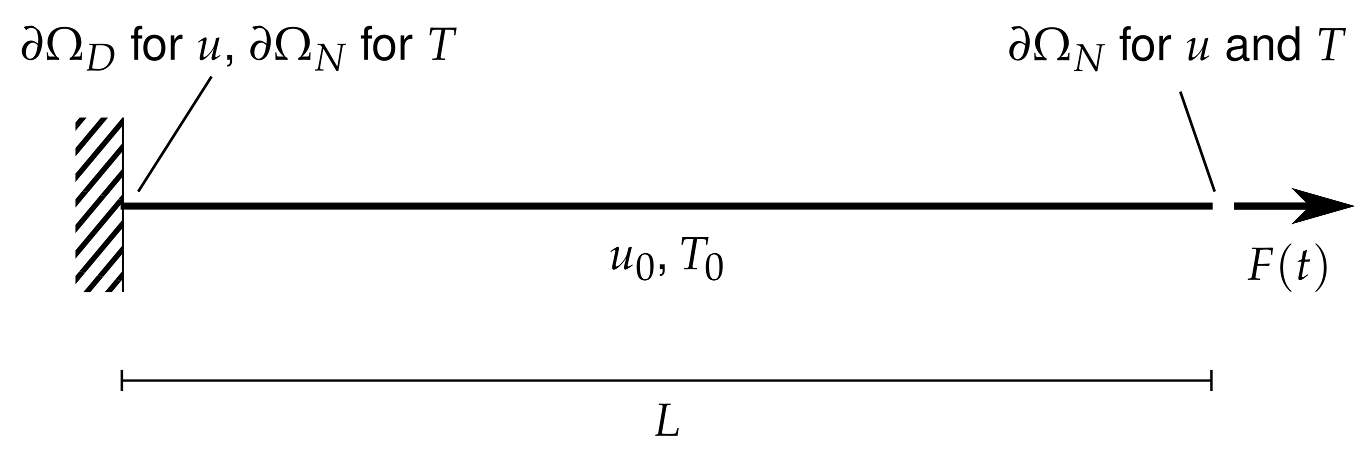
## Model Problem

Analysis of boundary value problem of linear thermoelastic and linear thermoviscoelastic material behavior of a one-dimensional bar element with length $L = 1\text{m}$.




### Linear thermoelastic material behavior

- Linear balance of momentum:

$$\rho\ddot{u} = Eu'' - \beta(T - T_0)'$$

- Heat conduction equation in full coupling:

$$\rho c\dot{T} = \rho r + kT'' - T\beta\dot{\varepsilon}$$

### Linear thermoviscoelastic material behavior

- Linear balance of momentum:

$$\rho\ddot{u} = Eu'' - \beta(T - T_0)' + \frac{1}{3}(\mu_1 + 2\mu_2)\,\dot{\varepsilon}'$$

- Heat conduction equation:

$$\rho c\dot{T} = \rho r + kT'' - T\beta\dot{\varepsilon} + \frac{1}{3}(\mu_1 + 2\mu_2)\,\dot{\varepsilon}^2$$

### Basics of thermoviscoelastic material model

- Mechanical stress: $\sigma = \sigma^e + \sigma^d$ (cf. Abali [1])
- Elastic stress: $\sigma^e = Eu' - \beta(T - T_0)$
- Dissipation stress: $\sigma^d = \frac{1}{3}(\mu_1 + 2\mu_2)\dot{\varepsilon}$ with viscoelastic material parameters $\mu_1$ and $\mu_2$ (cf. Abali [1])

## Numerical Model

- Spatial discretization: Bubnov-Galerkin FEM with isoparametric concept
- Temporal discretization: Implicit Midpoint Rule

## Simulation Results

Simulation of linear thermoelastic and thermoviscoelastic material behavior for a starting deflection of $0.1\text{m}$ evenly distributed across bar element. No external energy is added to the system.

Dirichlet boundary $\partial\Omega_D$:

$$u(x = 0, t) = \bar{u} = 0$$

Neumann boundary $\partial\Omega_N$:

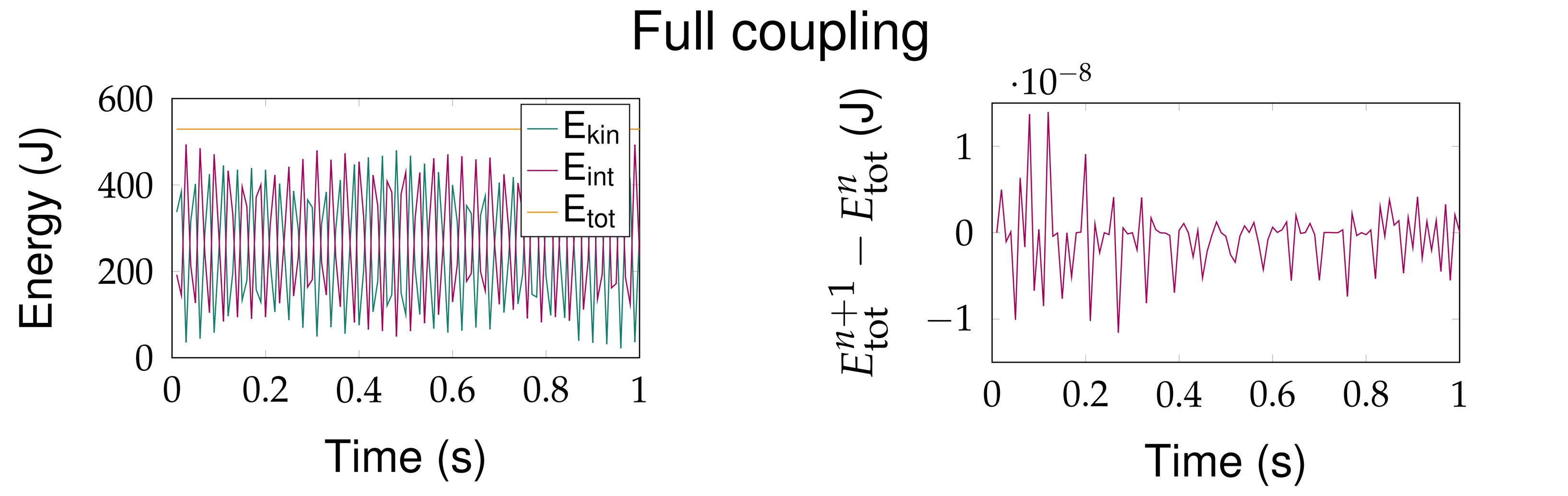
$$\begin{aligned} u'(x = L, t) &= F(t) &= 0,\\ T'(x = 0, t) &= T_0'(t) &= 0,\\ T'(x = L, t) &= T_0'(t) &= 0 \end{aligned}$$

Initial values: $T_0 = 293.15\text{K}$, $u_0(L) = 0.1\text{m}$

### Linear thermoelastic material behavior

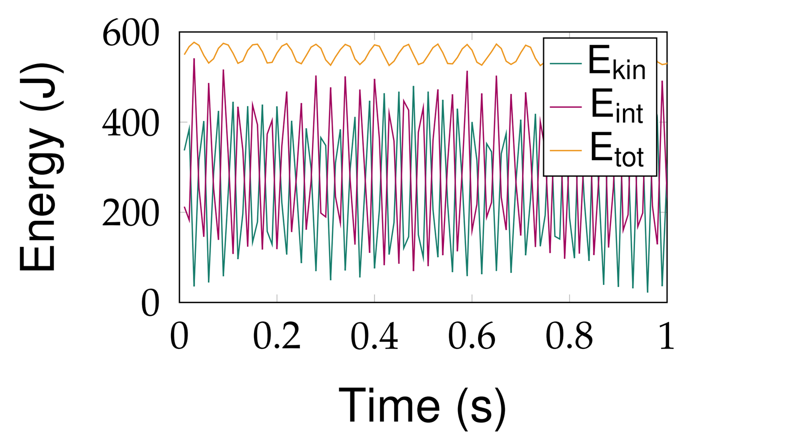
- For heat conduction equation in full coupling, energy conserving formulation can be achieved
- Energy difference between time steps consistently below Newton tolerance of $\texttt{eps} = 10^{-6}$

Full coupling

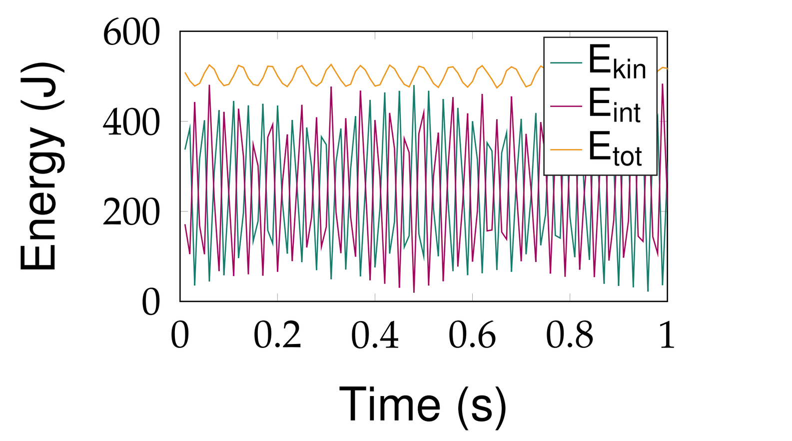



- Simplified coupling ($T\beta\dot{\varepsilon} \to T_0\beta\dot{\varepsilon}$) and weak coupling ($T\beta\dot{\varepsilon} \to 0$) of heat conduction equation: energy conservation not possible

Simplified coupling

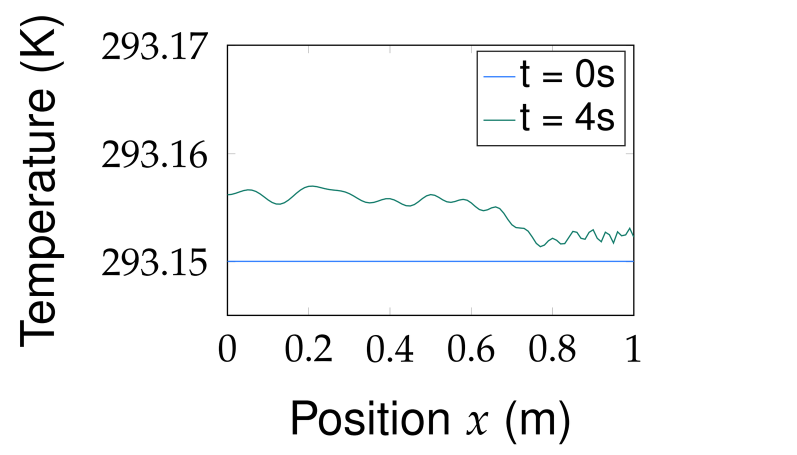



Weak coupling

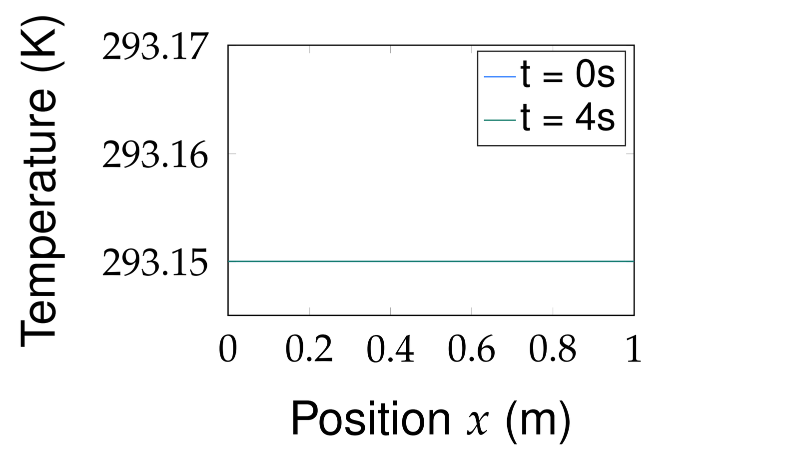



- Heat conduction equation in full coupling: deformation ⇒ temperature change
- Weak coupling: deformation has no effect on temperature

Full coupling

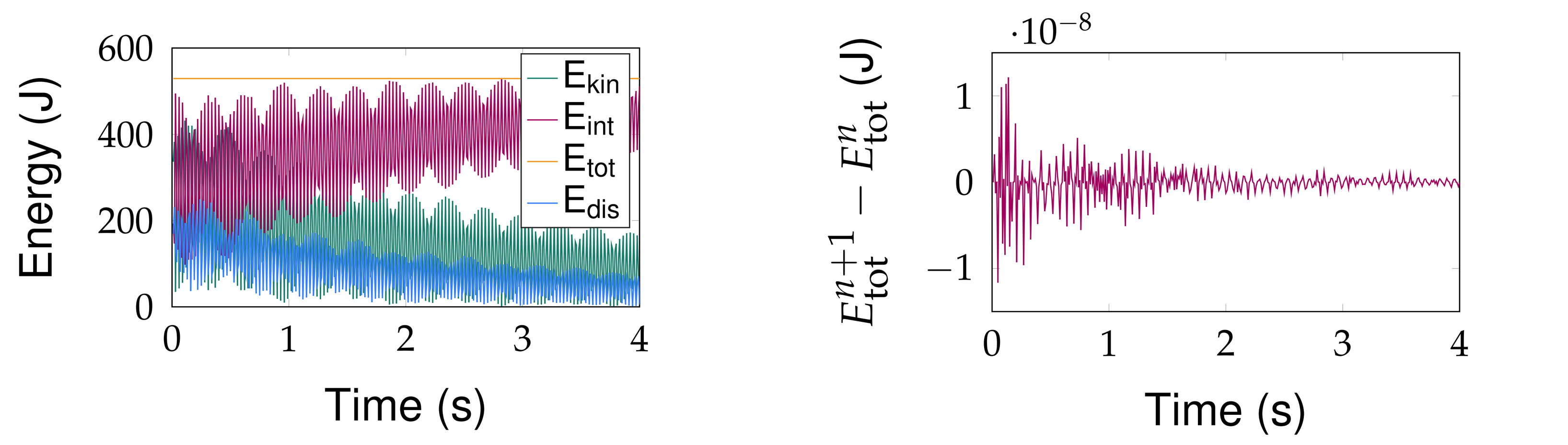



Weak coupling




### Linear thermoviscoelastic material behavior

- Energy conserving formulation is achieved for full coupling
- Effect viscoelasticity: kinetic energy transforms into internal energy in form of heat ⇒ displacement is dampened, temperature increases




## References

[1] Abali, B. E. *Computational Reality-Solving Nonlinear and Coupled Problems in Continuum Mechanics*. Singapore: Springer Singapore, 2017.

[2] Hetnarski, R. B. and Eslami, M. R. *Thermal Stresses—Advanced Theory and Applications*. Cham, Switzerland: Springer Cham, 2019.