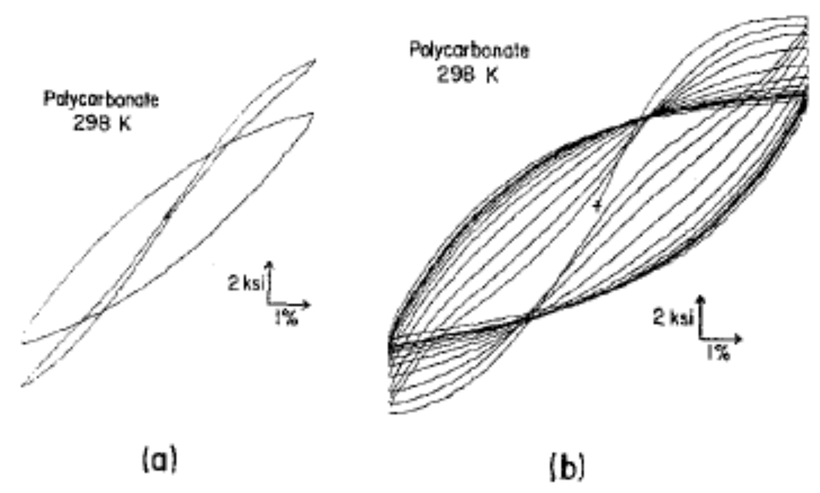
Institute of Mechanics
Prof. Dr.-Ing. habil. Peter Betsch
Prof. Dr.-Ing. habil. Thomas Seelig

Modelling cyclic softening with the viscoelastic Poynting-Thomson model

Pit Reiff | Bachelor Thesis (2020)

Introduction

In civil engineering, a common material model is Hooke's law. However, many materials do not fit the assumptions of Hooke's law of a linear relation between stress and strain, as it is neglecting deformations and time-dependent (viscoelastic) phenomena like creep and relaxation which are observed in materials like polymers. A model which is capable of describing such phenomena is the standard model of viscoelasticity, also known as the Poynting Thomson (PT) model (Fig. right). In engineering practice, the being aware of the dangers of material fatigue is crucial for safety reasons, especially for structures subjected to cyclic loading, where cyclic softening can be observed (Fig. left).



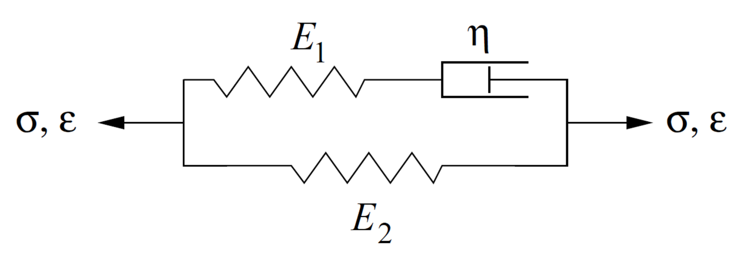


Figure 1: *Cyclic softening in polycarbonate (left) [1] and Poynting Thomson model [2] (right)*

- Cyclic softening: continuous decrease of stiffness under cyclic loading
- Objective: numerical modelling of cyclic loading based on PT model, extension of the PT model to incorporate softening and carrying out a parameter study to evaluate the model.

Initial value problem (IVP)

ODE $\dot{\sigma} + \frac{E_1}{\eta}\sigma = (E_1 + E_2)\,\dot{\varepsilon} + \frac{E_1\,E_2}{\eta}\varepsilon$

dissipated work $\dot{W} = \frac{1}{\eta}(\sigma - E_2\,\varepsilon)^2$

degraded parameter $P_i = P_0 \exp\left(-\frac{r}{E_{ref}\,\varepsilon_0^2}\cdot W\right) + P_\infty$

with $P_i \in \{E_2; E_1; \eta\}$

$\varepsilon(t) = \varepsilon_m + \varepsilon_0 \sin(\omega\,t)$

$\dot{\varepsilon}(t) = \varepsilon_0\,\omega\,\cos(\omega t)$

I.C. $\sigma(t=0) = 0$

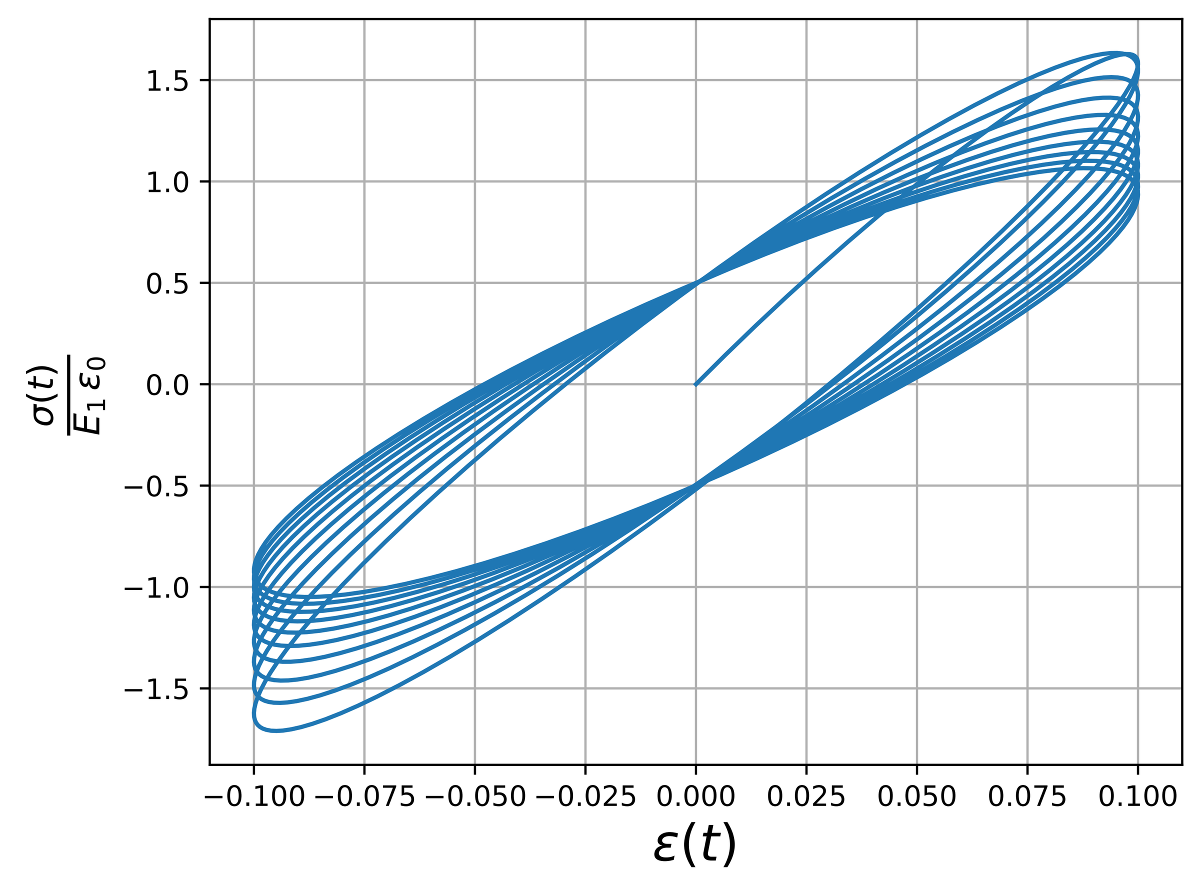
- Analytical solution (without softening)

$$\Rightarrow \sigma(t) = (-E''\,\varepsilon_0 - \varepsilon_m)\,e^{-\frac{t}{\tau}} + (E''\,\varepsilon_0)\cos(\omega\,t) + (E'\,\varepsilon_0)\sin(\omega\,t) + E_2\,\varepsilon_m$$

  where E' and E'' are the storage/loss moduli.
- Different numerical discretization methods are compared: the forward Euler method, the backward Euler method as well as the standard Runge-Kutta method.

Results

- Parameter study: analysis of the influence of the different parameters: load ($\omega\tau$), degradation parameter r and the degraded parameters P_i
- Examplary model response for variation of Young's modulus E_2 with $\omega\tau = 1$ and $r = 0.1$:



Conclusion and outlook

- Three possibilities to describe cyclic softening in PT model: degradation of E_1, E_2, η, but with very different numerical results !
- Outlook
  - comparison to experiments in order to determine the model quality (degradation of E_2 looks the most sensible)
  - degradation of P_i as a function of temperature

References

[1] Rabinowitz, S. and Beardmore, P. Cyclic deformation and fracture of polymers. In: *Journal of Materials science*, (9): 81–99, 1974.

[2] Seelig, T. *Anwendungsorientierte Materialtheorien*. 2016.