Design of a Novel Petrov-Galerkin Finite Element Formulation for the Simulation of Plate Structures

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Abstract

In this work, within the framework of the Mindlin-Reissner plate theory, a novel Petrov-Galerkin Enhanced Assumed Strain (EAS) element formulation is developed. For this purpose, the Petrov-Galerkin method is employed, in which the test and trial functions are approximated differently. The foundation for the novel formulation is the EAS plate element by Simo and Rifai [3], as well as the Petrov-Galerkin EAS formulation for two-dimensional elasticity by Pfefferkorn and Betsch [2]. The combination of both methods is developed and investigated concerning its ability to produce a mesh-independent formulation.

EAS for Plates by Simo and Rifai

- Degrees of freedom and displacement vector
  \( \mathbf{u} = \begin{bmatrix} w \\ \Theta \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta_x \\ \beta_y \end{bmatrix} \)

- Kinematics
  \( \begin{bmatrix} \kappa \\ \gamma \end{bmatrix} = \begin{bmatrix} \nabla \Theta \\ \nabla w \end{bmatrix} \)

- Hu-Washizu 3-field formulation
  \( \Pi_{\text{ex}}(u, \tau, \gamma) = \int_{\Omega} \left( \frac{1}{2} \kappa^T E \kappa \, dA + \frac{1}{2} \gamma^T \tilde{E} \gamma \, dA \right) + \int_{\Omega} \left( \tau^T \left( \nabla w - \Theta - \gamma \right) \right) \, dA \)

- Enhancement of the shear strain
  \( \gamma = \nabla w - \Theta + \gamma \)

- Equilibrium
  \( \delta (\Pi_{\text{int}} + \Pi_{\text{ex}}) = 0 \)

Petrov-Galerkin Finite Element Discretization

- Basic idea: different shape functions for test and trial functions
- Lagrangian shape functions for the test function
- Metric shape functions in skew coordinates for the trial function according to Xie et al. [4]
- Discrete enhanced shear strain and its virtual counterpart
  \( \gamma_j^e(\xi) = \frac{\partial}{\partial \xi_j} J_0^T \tilde{E}(\xi) \delta \alpha = H \delta \alpha \)

Elimination of the Locking Modes

- Problem: transverse shear locking for distorted meshes
- Locking modes have polynomial form
- Conditions: patch test and orthogonality of shear stress and enhanced shear strain → use of Legendre polynomials as basis for interpolation matrices for the enhanced shear strain
- Locking can be eliminated with the right choice for \( E \) and \( E \)
- Resulting stiffness matrix depends on nodal coordinates → strong mesh dependency expected

Mesh Distortion Test

- Example of a simply supported rectangular plate with an constant area load
- Three nodes are shifted at angles of 45°
- Strong mesh sensitivity as expected
- Same result as ANS element of Bathe and Dvorkin [1] for regular mesh \((s = 0)\)
- Conclusion: mesh independence cannot be achieved

References