Karlsruhe Institute of Technology

Institute of Mechanics
Prof. Dr.-Ing. habil. Peter Betsch
Prof. Dr.-Ing. habil. Thomas Seelig

Design of a Novel Petrov-Galerkin Finite Element Formulation for the Simulation of Plate Structures

Pit Reiff | Master Thesis (2023)

Abstract

In this work, within the framework of the Mindlin-Reissner plate theory, a novel Petrov-Galerkin Enhanced Assumed Strain (EAS) element formulation is developed. For this purpose, the Petrov-Galerkin method is employed, in which the test and trial functions are approximated differently. The foundation for the novel formulation is the EAS plate element by Simo and Rifai [3], as well as the Petrov-Galerkin EAS formulation for two-dimensional elasticity by Pfefferkorn and Betsch [2]. The combination of both methods is developed and investigated concerning its ability to produce a mesh-independent formulation.

EAS for Plates by Simo and Rifai

- Degrees of freedom and displacement vector

$$\mathbf{u} = \begin{bmatrix} w \\ \boldsymbol{\Theta} \end{bmatrix} = \begin{bmatrix} w \\ \beta_x \\ \beta_y \end{bmatrix}$$

- Kinematics

$$\begin{bmatrix} \boldsymbol{\kappa} \\ \boldsymbol{\gamma} \end{bmatrix} = \begin{bmatrix} \nabla^{\mathrm{S}}\boldsymbol{\Theta} \\ \nabla w - \boldsymbol{\Theta} \end{bmatrix}$$

- Hu-Washizu 3-field formulation

$$\Pi_{\text{int}}(\mathbf{u}, \boldsymbol{\tau}, \boldsymbol{\gamma}) = \int_\Omega \frac{1}{2}\boldsymbol{\kappa}^{\mathrm{T}}\mathbf{E}_{\mathbf{B}}\boldsymbol{\kappa}\,\mathrm{d}A + \int_\Omega \frac{1}{2}\boldsymbol{\gamma}^{\mathrm{T}}\mathbf{E}_{\mathbf{S}}\boldsymbol{\gamma}\,\mathrm{d}A + \int_\Omega \left(\boldsymbol{\tau}^{\mathrm{T}}\left(\nabla w - \boldsymbol{\Theta} - \boldsymbol{\gamma}\right)\right)\mathrm{d}A$$

- Enhancement of the shear strain

$$\boldsymbol{\gamma} = \nabla w - \boldsymbol{\Theta} + \tilde{\boldsymbol{\gamma}}$$

- Equilibrium

$$\delta\left(\Pi_{\text{int}} + \Pi_{\text{ext}}\right) \stackrel{!}{=} 0$$

Petrov-Galerkin Finite Element Discretization

- Basic idea: different shape functions for test and trial functions
- Lagrangian shape functions for the test function
- Metric shape functions in skew coordinates for the trial function according to Xie et al. [4]
- Discrete enhanced shear strain and its virtual counterpart

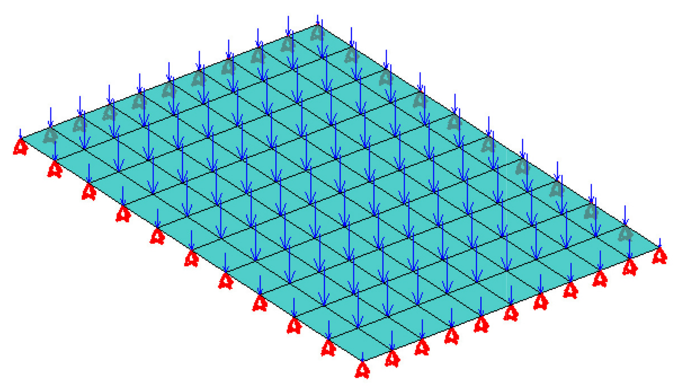
$$\tilde{\boldsymbol{\gamma}}_h^e(\boldsymbol{\xi}) = \frac{j_0}{j}\mathbf{J}_0^{-\mathrm{T}}\tilde{\mathbf{E}}(\boldsymbol{\xi})\boldsymbol{\alpha} = \mathbf{H}\boldsymbol{\alpha}$$
$$\delta\tilde{\boldsymbol{\gamma}}_h^e(\boldsymbol{\xi}) = \frac{j_0}{j}\mathbf{J}_0^{-\mathrm{T}}\mathbf{E}(\boldsymbol{\xi})\delta\boldsymbol{\alpha} = \mathbf{G}\delta\boldsymbol{\alpha}$$

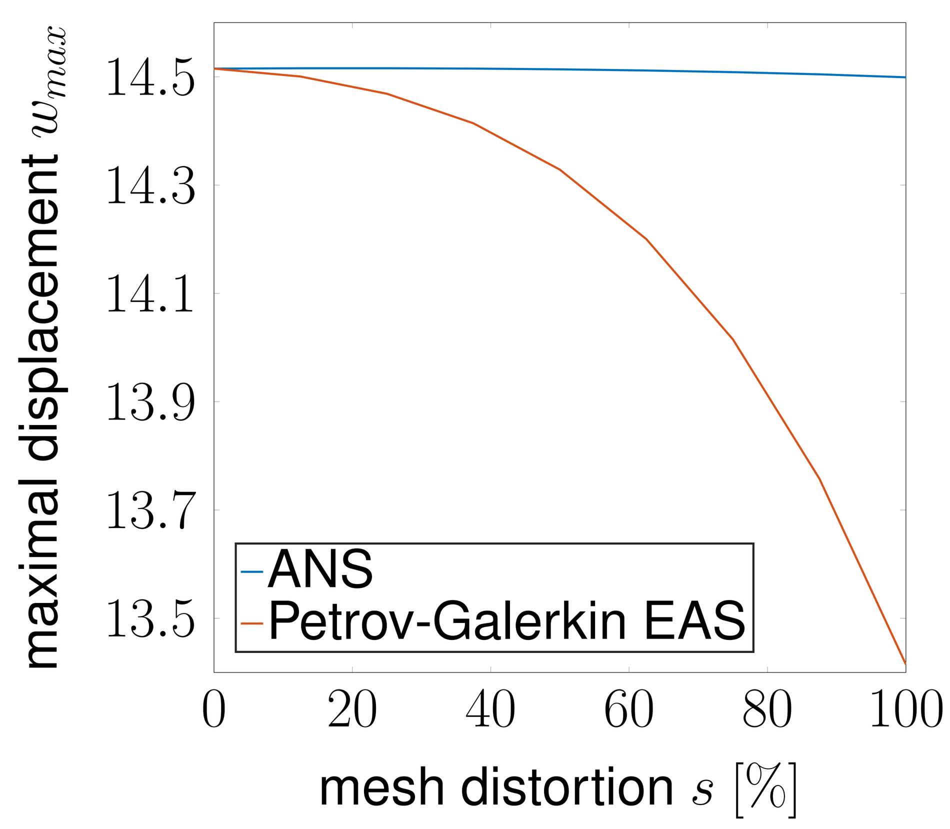
Elimination of the Locking Modes

- Problem: transverse shear locking for distorted meshes
- Locking modes have polynomial form
- Conditions: patch test and orthogonality of shear stress and enhanced shear strain → use of Legendre polynomials as basis for interpolation matrices for the enhanced shear strain
- Locking can be eliminated with the right choice for $\mathbf{E}$ and $\tilde{\mathbf{E}}$
- Resulting stiffness matrix depends on nodal coordinates → strong mesh dependency expected

Mesh Distortion Test

- Example of a simply supported rectangular plate with an constant area load





- Three nodes are shifted at angles of 45°
- Strong mesh sensitivity as expected
- Same result as ANS element of Bathe and Dvorkin [1] for regular mesh ($s = 0$)
- Conclusion: mesh independence cannot be achieved

References

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