Modeling of inflatable dams partially filled with fluid and gas considering large deformations and stability

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Anne Maurer Marc Hassler Karl Schweizerhof

Institute of Mechanics
Kaiserstr. 12
D-76131 Karlsruhe
Tel.: +49 (0) 721/ 608-2071
Fax: +49 (0) 721/608-7990
E-Mail: info@ifm.kit.edu
www.ifm.uni-karlsruhe.de

1 SUMMARY

Although gas or fluid supported membrane structures have a wide field of applications, such as inflatable pontoons of pioneers, hover-craft cushions or even pressurized girders or dams in the field of structural engineering, for fluid-structure interaction algorithms it is still a challenge to manage problems with large structural deformations in combination with gas and/or fluid containments changing their inner state variables during the deformation. In order to describe such interaction problems an analytical meshfree description for the fluid/gas is chosen [1], [3]. This approach leads to a deformation dependent loading and the particular advantage that concerning stability the influence of internal gas and fluid filling can be directly computed [4].
The deployment of an inflatable dam from partial filling to complete filling with fluid and gas followed by a hydrostatic loading demonstrates, how the simulation of structures with static gas and fluid loading resp. support can be efficiently performed for large deformations without discretizing the fluid resp. the gas. A stability investigation shows, how far such a dam has to be filled with fluid to avoid major stability problems. Also multi-chamber systems are presented as a further alternative and the efficiency of different solution schemes is discussed.

2 INTRODUCTION

In this contribution structures filled and loaded with fluid and/or gas undergoing large deformations will be analysed with respect to their static stability and the discussion of different solution schemes. As already shown in [5] and [6] the analytical formulation of the fluid/gas terms in statics depends only on the surrounding structure and some fundamental variables as e.g. the gas pressure. The derivations in [4] lead to a linearized set of equations for fluid and/or gas filled chambers, which will be briefly discussed. Further, the focus is on the terms, which are important for the specific solution schemes and for the stability. The numerical examples cover both the buckling analyses of air inflated flexible dams under hydrostatic loading and a comparison of iterative solvers and direct solvers used for the computation.

3 FINITE ELEMENT FORMULATION OF GAS FILLED STRUCTURES

As the derivation of the equations describing the state of equilibrium of a structure filled with fluid and/or gas only in terms of variables of the structural geometry are already given in [4], in this section only the basic approach for a system filled with gas will be shown. All other cases, as e.g. the filling with compressible fluid, incompressible fluid and incompressible fluid with additional gas loading, are treated in a similar fashion. The principle of virtual work states that for a structure at equilibrium subjected to a virtual displacement field the variation of the total potential energy $\delta\Pi$ equals the virtual work of the external forces δW_{ext},

$$\delta\Pi - \delta W_{ext} = 0. \tag{1}$$

Considering a structure filled with gas, the energy consists of two parts: the internal energy Π_{el} of the elastic structure and the internal energy Π_g stored in the gas. For the virtual change of the energy then follows

$$\delta\Pi = \delta\Pi_{el} + \delta\Pi_g. \tag{2}$$

The equilibrium is considered in the current configuration Φ at time t. Applying the Galerkin method to the surrounding structure results in disjunct elements and basis functions $d = N \cdot u$ for the displacements u. Inserting (2) in (1) and linearizing the equations leads for $\delta\Pi_g$ to terms corresponding to a pressure change Δp and a change Δn of the normal n on the surrounding structure,

$$\delta\Pi_g(\Phi + \Delta u) = \underbrace{\delta\Pi_g(\Phi)}_{(a)} + \underbrace{\frac{\partial \delta\Pi_g^{\Delta p}(\Phi)}{\partial u}\Delta u}_{(b)} + \underbrace{\frac{\partial \delta\Pi_g^{\Delta n}(\Phi)}{\partial u}\Delta u}_{(c)}. \tag{3}$$

The residual term (a) only depends on the virtual displacement δu and thus it belongs to the right-hand side vector f_g. The second term (b) depends on δu and the change of pressure Δp and will appear as an additional column a to the assembled stiffness matrix. The matrix K_g which results of the term (c) and depends on δu and the change of displacement Δu will be added to the stiffness matrix K_{el} of the system. The added column, leading to a rank-one-update in a pure displacement formulation, is responsible for a certain stabilization of the system. Altogether this approach leads, with the discretization of the structure and using basis functions for the displacements, to the first row in the following system of equations,

$$\begin{pmatrix} K_{el} + K_g & -a \\ -a^T & -\alpha \end{pmatrix} \begin{pmatrix} \Delta d \\ \Delta p \end{pmatrix} = \begin{pmatrix} f_{el} + f_g \\ 0 \end{pmatrix}. \tag{4}$$

The second row follows from the isothermal constitutive equation for a quasistatic volume change. This system can be solved with an iterative solver like the SQMR method or with a direct solver. If the system consists of more than one chamber, the assembled matrix in (4) features further rank updates. The choice of the fastest solution scheme depends on the structure of this matrix. Also the challenge how to handle the very flexible state - almost kinematic - at the beginning of the filling process numerically will be discussed.

4 NUMERICAL EXAMPLES

An example for structures filled with gas and undergoing large deformations are inflatable dams [2]. They consist of a rubber coated tube with composite layers which is fixed to the floor. The hydrostatic loading can be head water and/or bottom water (Figure

1). The focus is here on the filling process and head water loading which causes large deformations and raises the problem of structural stability.

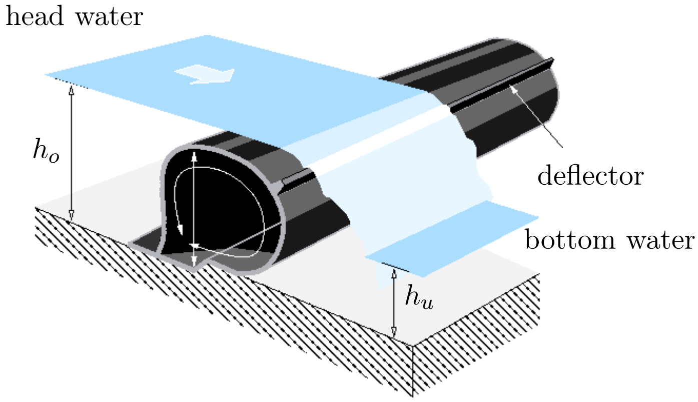

Figure 1: Layout and load of an inflatable dam

For the filling process the uncoiled flexible tube is subjected to an increasing internal gas pressure, inflating the membrane without any hydrostatic loading. In a further step the head water level is steadily increased until it reaches the weir crest. This simulates a rubber dam with hydrostatic loading and without overflow.

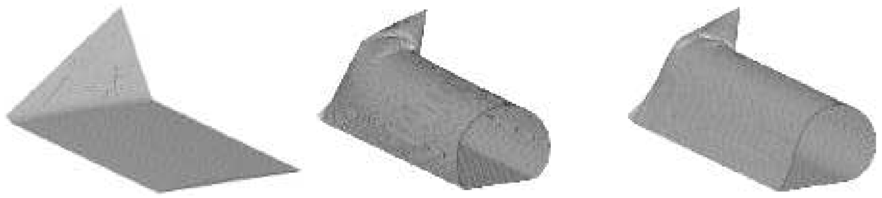

Figure 2: Initial configuration, filling process, hydrostatic load simulated by finite elements

As air filled rubber dams are prone to instability in case of high head water leading to buckling and to a localised overflow, they can be stabilized with an additional fluid filling. How far is subject of the presented investigation.

Figure 3: Buckling due to overflow

References

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