Simulation of Fast Crack Growth in Brittle Materials

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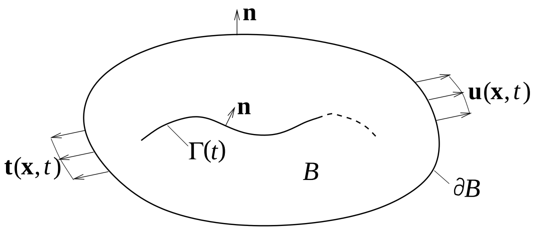
Problem Setting

- precracked body under dynamic loading conditions
- trajectory and speed of crack growth a priori unknown



Mathematical Formulation

- boundary integral equations (BIE) of elastodynamics

$$c_{kl}(\mathbf{x})\,u_k(\mathbf{x},t) + \int_{\partial B} \{ n_j\,\sigma^G_{ijl} * u_i - u^G_{il} * t_i \}\, dA(\mathbf{y})$$

$$= \int_{\Gamma(t)} n_j\,\sigma^G_{ijl} * \Delta u_i\, dA(\mathbf{y}) \ , \qquad \mathbf{x} \in \partial B$$

$$C_{pqlk}\,n_q(\mathbf{x}) \int_{\Gamma(t)} \{ \sigma^G_{ijl} * (n_k\partial_j - n_j\partial_k)\Delta u_i + \rho\, n_k\, u^G_{il} * \Delta\ddot{u}_i \}\, dA(\mathbf{y})$$

$$= C_{pqlk}\,n_q(\mathbf{x}) \int_{\partial B} \{ \sigma^G_{ijl} * (n_k\partial_j - n_j\partial_k)\, u_i + \rho\, n_k\, u^G_{il} * \ddot{u}_i \}\, dA(\mathbf{y})$$

$$- n_q(\mathbf{x}) \int_{\partial B} \sigma^G_{pqi} * t_i\, dA(\mathbf{y}) \ - \ t^{\Gamma}_p(\mathbf{x},t) \ , \qquad \mathbf{x} \in \Gamma(t)$$

- 2D fracture criterion ('max. circum. stress')

$$F(\,K_I, K_{II}, \dot{\mathbf{a}};\, K_D(\dot{\mathbf{a}})\,) \le 0 \quad \Longrightarrow \quad \dot{\mathbf{a}} \ \text{(crack tip speed)}$$

K_I, K_{II} : dyn. stress intensity factors (from BIE)

$K_D(\dot{\mathbf{a}})$: dyn. fracture toughness (from experiments)

Numerical Treatment

- time-domain boundary element method (BEM)
- incremental crack growth ($\Delta\mathbf{a}$) by adding new elements

Literature

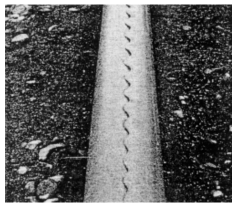
Th. Seelig, D. Gross (1999). On the stress wave induced curving of fast running cracks - a numerical study by a time-domain boundary element method. *Acta Mechanica* **132**, 47-61

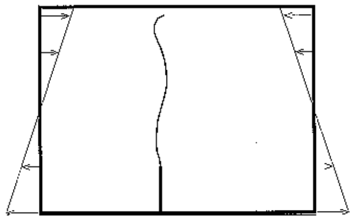
Th. Seelig, D. Gross (1999). On the interaction and branching of fast running cracks - a numerical investigation. *Journal of the Mechanics and Physics of Solids* **47**, 935-952

Th. Seelig, D. Gross and K. Pothmann (1999). Numerical simulation of a mixed-mode dynamic fracture experiment. *Int. J. Fract.* **99**, 325-338

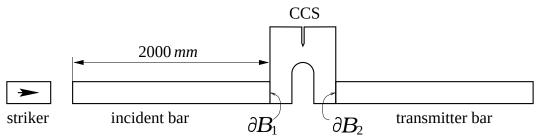
Results

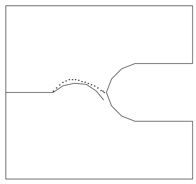
- influence of stress waves
  (repeated interaction with running crack may lead to oscillatory crack path)





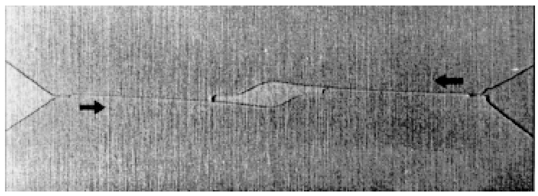
- simulation of dynamic fracture experiment
  ('compact compression specimen' in split Hopkinson bar)





— : BEM .... : exp. (by D. Rittel, Technion, Haifa)

- interacting cracks



(by S. Melin, 1983, *Int. J. Fract.* **23**)