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Vibration Analysis of Thin-Walled - Gas or Fluid Filled - Structures Including the Effect of the Inflation/Filling Process

Karl Schweizerhof*, Marc Haßler

Institut für Mechanik, Universität Karlsruhe (TH)
Englerstr. 2, D-76131 Karlsruhe, Germany
karl.schweizerhof@ifm.uni-karlsruhe.de

Abstract

Fluid-structure interaction problems involving thin-walled membrane and shell-type structures undergoing large deformations can be considered in a step-wise fashion. Using conventional finite elements [1],[6],[8] for the discretization of the fluid or gas domain leads to heavily distorted meshes for the fluid if e.g. inflation or filling processes including large deformations are to be investigated. Although with ALE based algorithms this problem can be solved by a permanent remeshing of the fluid domain parts in the vicinity of the structural mesh, this is at the expense of high computational effort. Benefitting from an algorithm, which replaces the fluid or gas filling by an energetically equivalent volume dependent surface loading (see e.g. [2]-[5] and [7]) such primarily static inflation processes can be simulated without discretizing the fluid or gas domain. Thus the deformation dependent inner state variables of the fluid or gas can be computed avoiding the previously mentioned difficulties with mesh distortions in an efficient way. In a following step the fluid parameters such as fluid level and information about the wetted structural parts can then be used to define properly the initial conditions for a dynamic finite element analysis of gas or fluid filled structures while using standard acoustic finite elements to compute e.g. vibration modes.

1 Virtual Work Approach of Gas or Fluid Loaded Structures

For a state of equilibrium in a system consisting of a fluid domain $\mathcal{F}$ and a solid domain $\mathcal{B}$ the variation $\delta\mathcal{E}$ of the total energy has to fulfill

$$\delta\mathcal{E} = \delta\mathcal{E}^{\mathcal{B}} + \delta\mathcal{E}^{\mathcal{F}} = 0 \; . \tag{1}$$

Assuming an adiabatic system, without any heat change $\delta\mathcal{Q}$, the energy conservation in the fluid and solid domain only consists of the variations $\delta\mathcal{T}^{\mathcal{B}\cup\mathcal{F}}$ and $\delta\mathcal{U}^{\mathcal{B}\cup\mathcal{F}}$ of the kinetic and the internal energy and of the virtual work $\delta\mathcal{W}^{\mathcal{B}\cup\mathcal{F}}$ of the external forces.

$$\delta\mathcal{E} = \delta\mathcal{T}^{\mathcal{B}\cup\mathcal{F}} + \delta\mathcal{U}^{\mathcal{B}\cup\mathcal{F}} - \delta\mathcal{W}^{\mathcal{B}\cup\mathcal{F}} = 0 \tag{2}$$

Introducing the virtual displacement field $\delta\boldsymbol{u}$, the accelerations $\ddot{\boldsymbol{u}}$ and the density ρ, the variation of the total kinetic energy can be written as

$$\delta\mathcal{T}^{\mathcal{B}\cup\mathcal{F}} = \int\limits_{\mathcal{B}\cup\mathcal{F}} \rho\ddot{\boldsymbol{u}} \cdot \delta\boldsymbol{u} \; dv \; . \tag{3}$$

The stress tensor $\boldsymbol{\sigma}$ and the virtual strains $\boldsymbol{\varepsilon}$ lead to the variation of the internal energy:

$$\delta\mathcal{U}^{\mathcal{B}\cup\mathcal{F}} = \int\limits_{\mathcal{B}\cup\mathcal{F}} \boldsymbol{\sigma} : \delta\boldsymbol{\varepsilon} dv \ . \tag{4}$$

Hence, the weak form (2) can be used for a finite element formulation to describe the fluid-structure interaction. The fact that inflation processes usually imply large deformations, which would lead to a high computational effort, if the fluid was disctretized with finite elements, in the following section an alternative way is briefly presented, using an analytical meshfree description of the fluid domain.

1.1 Quasi static inflation

The goal of the approach [3] is to replace the fluid by an energetically equivalent surface load, assuming that the fluid can be understood as a one-phase system, which means it is fully described by its inner state variables as e.g. pressure p or density ρ. As the detailed derivation for an arbitrary combination of gas and/or fluid loaded shell structures can be found in [3] this section contains only the basic idea of the algorithm for the example of a pneumatic single degree of freedom system. For the general case the finite element set of equations is of the same structure. However, considering the fluid as a one-phase system with only a few degrees of freedom simulations are limited to quasi static processes, because no information of acoustic wave propagation inside the fluid can be provided. For the quasi static case the variation of the kinetic energy can be neglected in the energy conservation (2).

$$\delta\mathcal{T}^{\mathcal{F}} = 0 \tag{5}$$

The fact that the fluid is a one-phase system can also be used to simplify the stress tensor to

$$\boldsymbol{\sigma} = -p\boldsymbol{I} \ , \tag{6}$$

with $\boldsymbol{I}$ as the unit tensor. After integrating over the fluid volume $\mathcal{F}$ and using

$$\boldsymbol{I} : \delta\boldsymbol{\varepsilon} = \mathrm{trace}(\delta\boldsymbol{\varepsilon}) = \delta v/v \tag{7}$$

the variation of internal energy of the fluid domain can be given as

$$\delta\mathcal{U}^{\mathcal{F}} = -p\delta v \tag{8}$$

and thus the total variation $\delta\mathcal{E}^{\mathcal{F}}$ of the potential energy of the fluid yields

$$\delta\mathcal{E}^{\mathcal{F}} = -p\delta v - \delta\mathcal{W}^{\mathcal{F}} \ . \tag{9}$$

1.1.1 1D example

We consider a closed system with rigid walls and one single degree of freedom u as depicted in figure 1. The elastic solid domain is represented by the spring with stiffness k and the fluid domain consists of an enclosed gas volume v with a pressure p. The system is subjected to an external force f^{ext}. For a state of equilibrium equation (2) yields with (5)

$$\delta\mathcal{E} = ku\delta u - p\delta v - f^{ext}\delta u = 0 \ . \tag{10}$$

For a given cross section A of the moving rigid wall the virtual volume change δv can be written in terms of the virtual displacement δu.

$$\delta v = A\delta u \tag{11}$$

Linearization of the equilibrium condition (10) at a current state t within a Newton scheme leads to

$$\delta\mathcal{E}_{lin} = (ku_t - p_tA - f^{ext})\delta u + k\Delta u\delta u + \Delta pA\delta u = 0 \ . \tag{12}$$

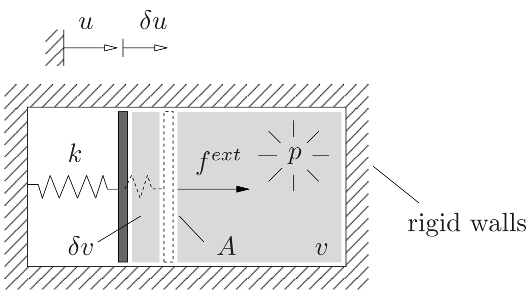

Figure 1: Virtual displacement δu applied to a pneumatic system with one degree of freedom u

Assuming isothermal behavior of the gas Boyle's law with $pv = const$ results in an equation for the incremental pressure change Δp, needed in equation (12):

$$\Delta p = -\frac{p_t}{v_t}\Delta v = -\frac{p_t}{v_t}A\Delta u \; . \tag{13}$$

Using equation (13) in (12) yields a fully displacement dependent formulation for the linearized state of equilibrium t with $u_{t+1} = u_t + \Delta u$.

$$\delta\mathcal{E}_{lin} = (ku_t - p_t A - f^{ext})\delta u + k\Delta u\delta u + \frac{p_t}{v_t}AA\Delta u\delta u = 0 \tag{14}$$

Eliminating the virtual displacement δu finally leads, along with the vector of the internal and external forces

$$f = f^{ext} - ku_t + p_t A \; , \tag{15}$$

to the equation for the unknown incremental displacement Δu.

$$\left(k + \frac{p_t}{v_t}AA\right)\Delta u = f \tag{16}$$

It can be seen that the volume dependence of the gas loading has both an influence on the stiffness and on the loading of the system. As already mentioned the general finite element formulation of a gas loaded shell structure (see e.g. [2], [4]) basically shows the same structure as equation (16) now with $\Delta\underline{d}$ as the nodal displacement vector: The stiffness matrix $\underline{K}$ is updated by a dyadic product of the discrete area vector $\underline{a}$ and the nodal force vector $\underline{f}$ is updated by the current pressure p_t.

$$\left(\underline{K} + \frac{p_t}{v_t}\underline{a}\underline{a}^T\right)\Delta\underline{d} = \underline{f} \tag{17}$$

For any arbitrary combination of fluid and/or gas loadings see [3].

1.2 Acoustic Fluid-Structure Interaction

After benefitting from the meshless but quasi static description of the fluid, the results obtained in the preceeding computation can serve as initial conditions in a subsequent dynamic analysis of the inflated and prestressed structure. Because in the vibration analysis the fluid mesh does not suffer from heavy distortions, it can be easily meshed with standard displacement dependent fluid elements, see e.g. [8] or [1], now allowing for kinetic energy changes (3). Discretizing both the position vector $\underline{X}_e$ and the displacement vector $\underline{u}_e$ of the fluid finite element with isoparametric shape functions $\underline{N}$, such that

$$\underline{X}_e = \underline{N}\hat{\underline{X}}_e \quad \text{and} \quad \underline{u}_e = \underline{N}\underline{d}_e \tag{18}$$

(with $\underline{\hat{X}}_e$ denoting the discrete nodal coordinates and $\underline{d}_e$ denoting the discrete nodal displacements), the kinematics resp. the volume changes of the element can be described. Introducing the matrix $\underline{B}$, the virtual and incremental volume changes on element level can be given as

$$\delta v = \int_{\mathcal{F}} \underline{B} \delta \underline{d}_e dv \quad \text{and} \quad \Delta v = \int_{\mathcal{F}} \underline{B} \Delta \underline{d}_e dv \tag{19}$$

and thus also the incremental pressure change

$$\Delta p = -\int_{\mathcal{F}} \frac{p_t}{v_t} \underline{B}_t \Delta \underline{d}_e dv \ , \tag{20}$$

necessary for the linearized weak form. Finally the linearized weak form of equilibrium at a current state t for a fluid element used for the subsequent vibration analysis follows as

$$\delta \mathcal{E}^{\mathcal{F}}_{lin} = \delta \mathcal{E}^{\mathcal{F}}_t + \delta \underline{d}_e{}^T \int_{\Omega_e} \rho_t \underline{N}^T \underline{N} dv \ \Delta \ddot{\underline{d}}_e + \delta \underline{d}_e^T \int_{\Omega_e} \frac{p_t}{v_t} \underline{B}^T \underline{B} dv \ \Delta \underline{d}_e - \delta \mathcal{W}^{\mathcal{F}} \ . \tag{21}$$

The third term in equation (21) is the analogon to the rank update in equation (17), with the difference that in (17) the energy of the fluid is described via the surrounding wetted surface $\partial\mathcal{B}$ of the solid domain and not by the volume integral over fluid elements as done in (21). Now an eigenvalue analysis is performed to find the eigenfrequencies and eigenmodes of the fluid filled an already highly deformed structure incorporating prestressing of the surrounding shell/membrane structure.

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