Microscopic modelling of open-cell foams

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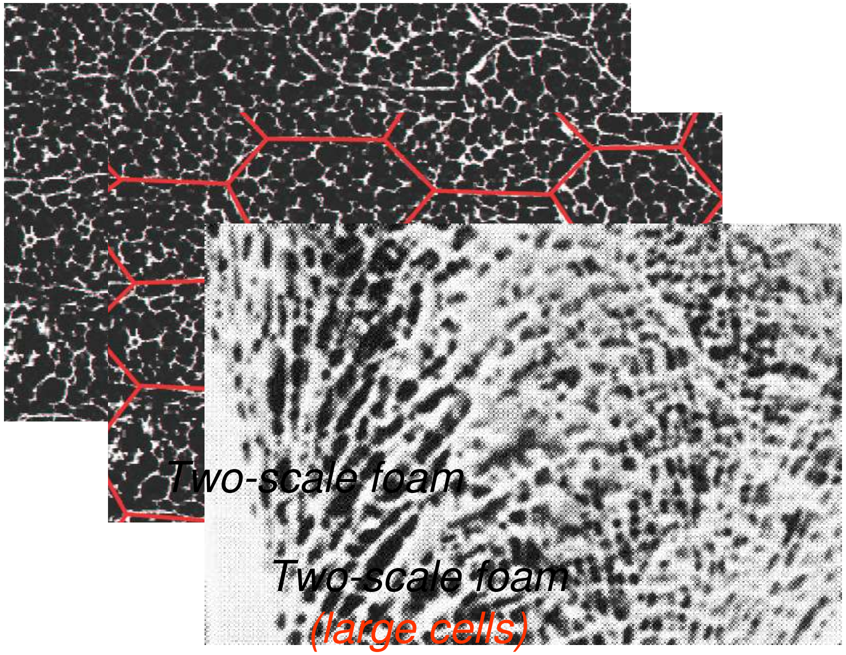
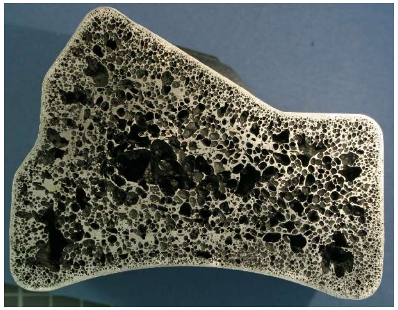
- **cellular materials**
  - polymer & metal foams
  - biological materials
    (hard tissues: spongiosa)
- **mechanical behaviour**
  - boundary layers under
    tension/compression
    bending and shear
  - size effects

→ **classical Boltzmann continua are not appropriate**



Femur: Spongiosa

observed size effects



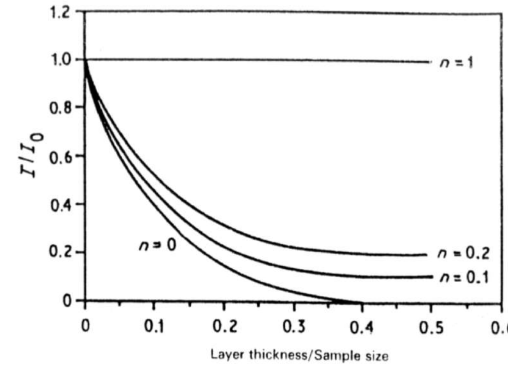


Figure 5 Relative moment of inertia, as predicted by the model, plotted against the ratio of the outer layer thickness and the base or height of the beam. Shown are curves for different values of $n = E_2/E_1$.

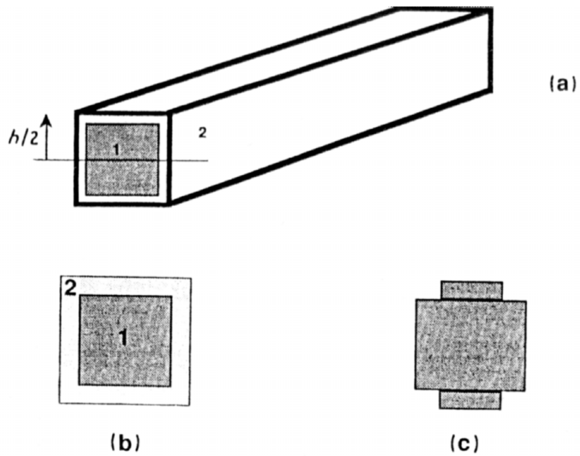


Figure 4 (a) Schematic of rectangular beam made up of two materials. (b) Unit cross section of beam having core of material 1 (modulus E_1) and an outer layer of material 2 (modulus E_2 where $E_2 < E_1$) having thickness X. (c) Transformed cross-section made up entirely of material 1 but maintaining the same resistance to bending as Fig. 4b.

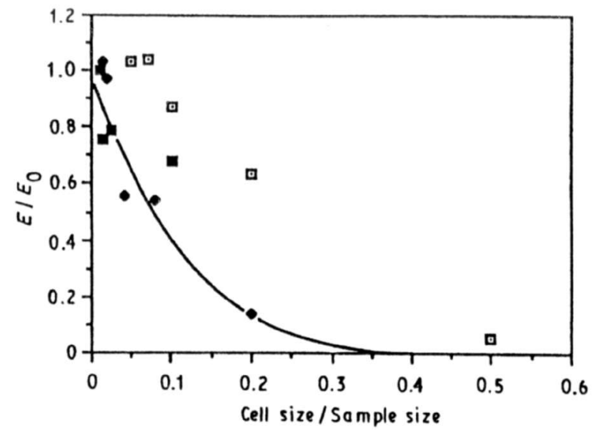
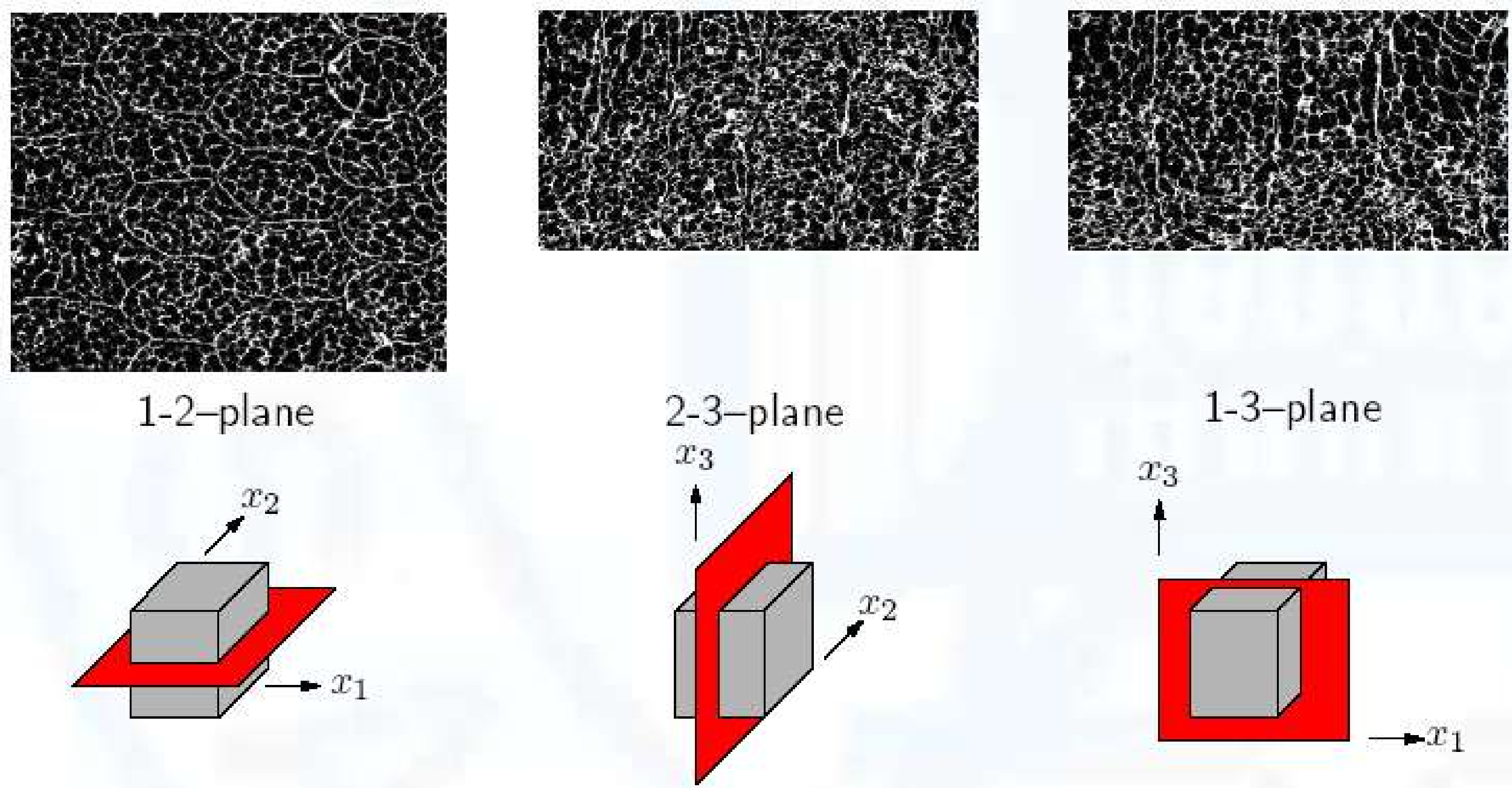


Figure 6 Experimental results of the three different cell size materials plotted as the relative elastic modulus against the ratio of cell size and the base or height of the samples. Also included is theoretical line given by model. (□ 2.5 mm, ◆ 0.56 mm, ■ 0.25 mm, ——— model.)

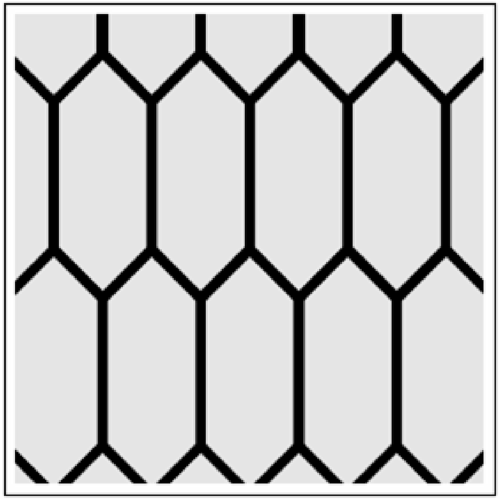
Brezny & Green *Journal of Materials Science* **25** (1990)

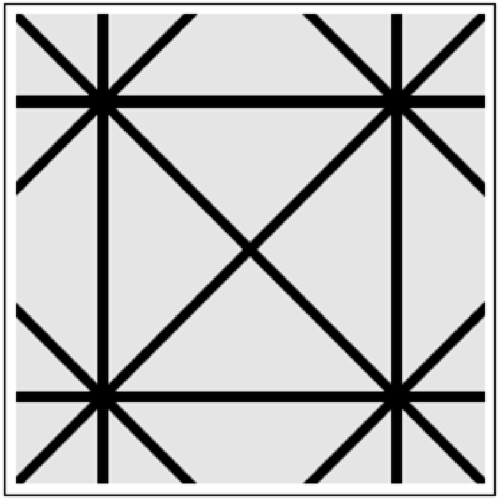
investigated microstructures: cellular topology

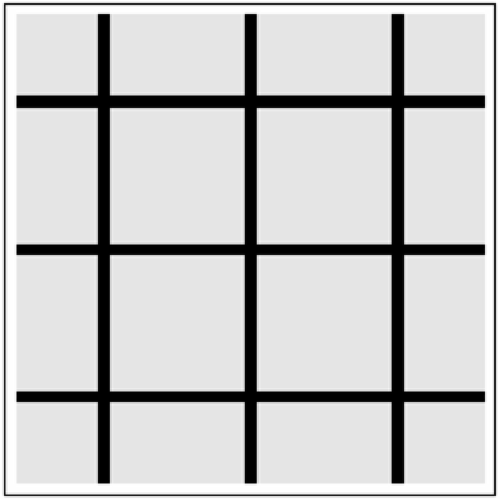
- pixel images from computer tomography (ct scan)



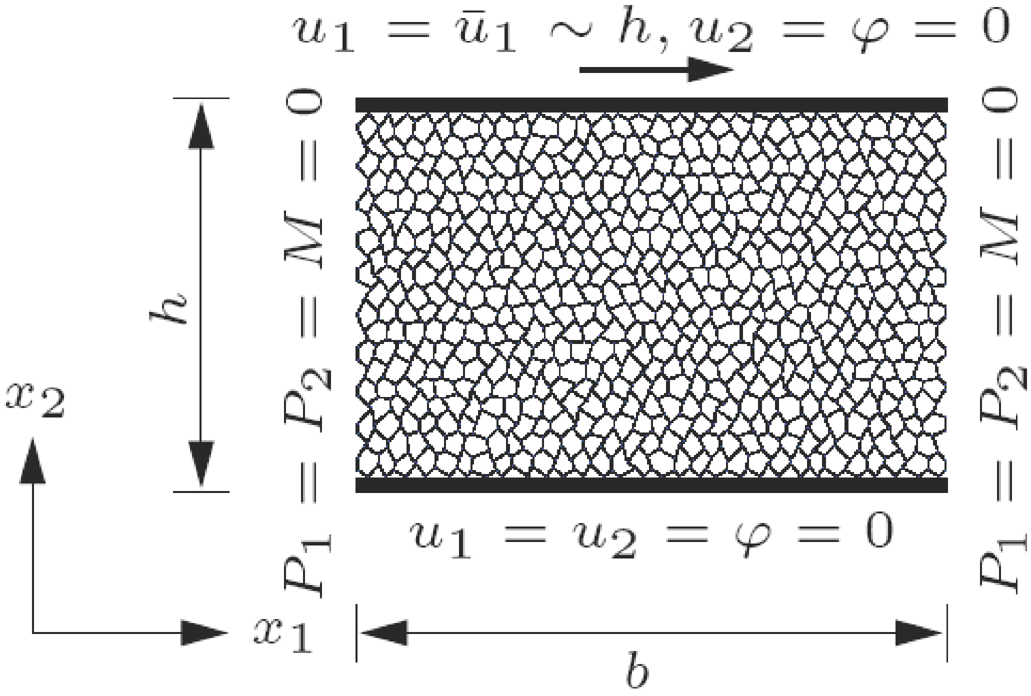
investigated microstructures: cellular topology

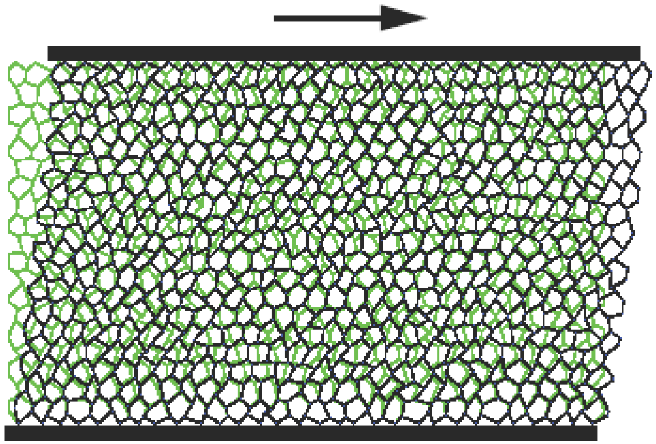




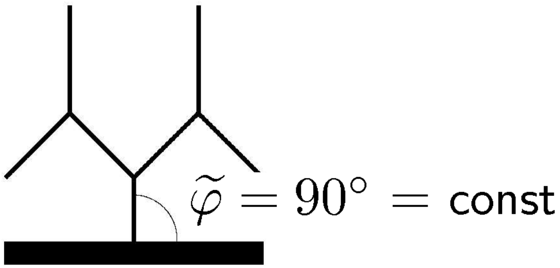


numerical experiments: shear test

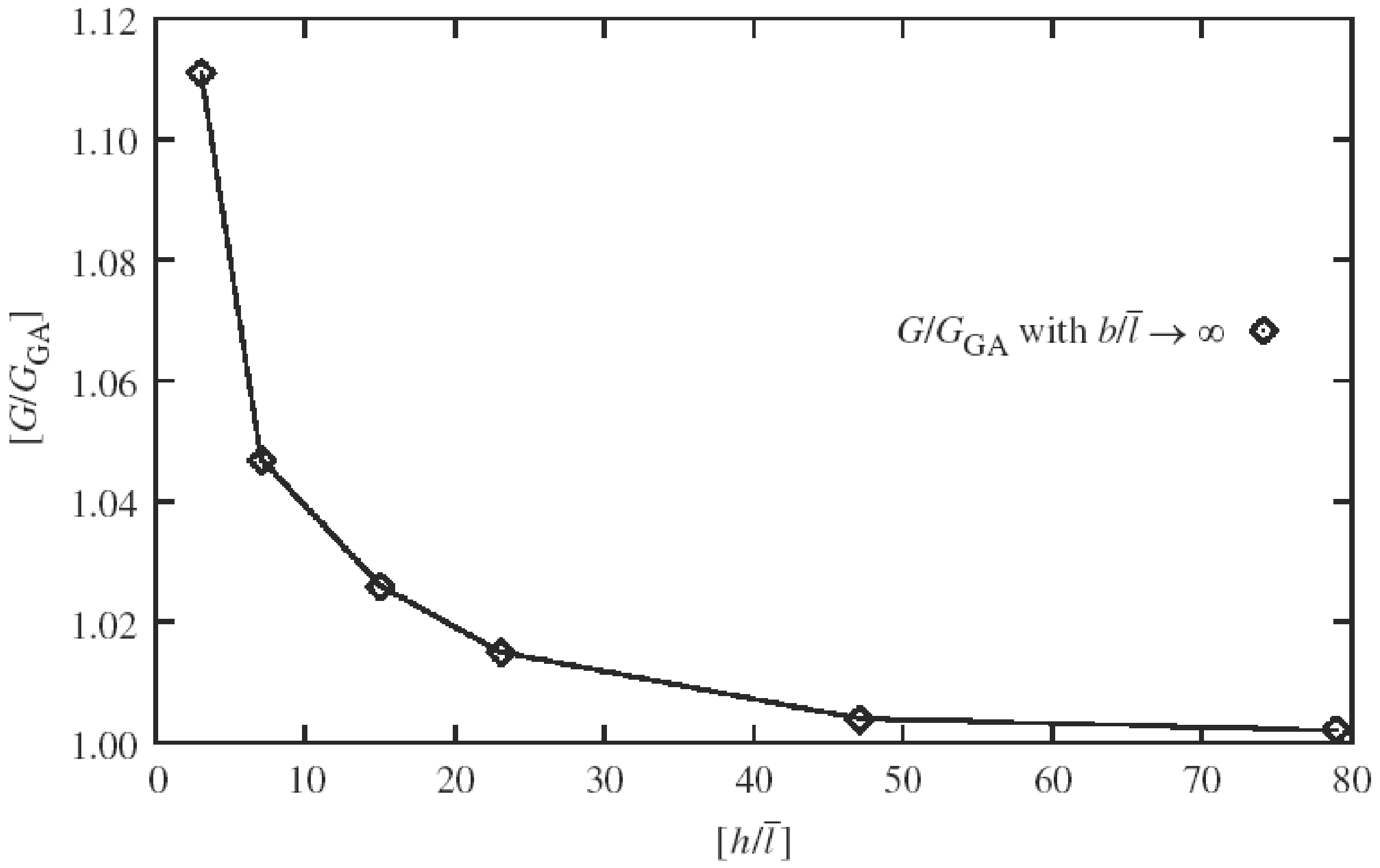




glued load platens



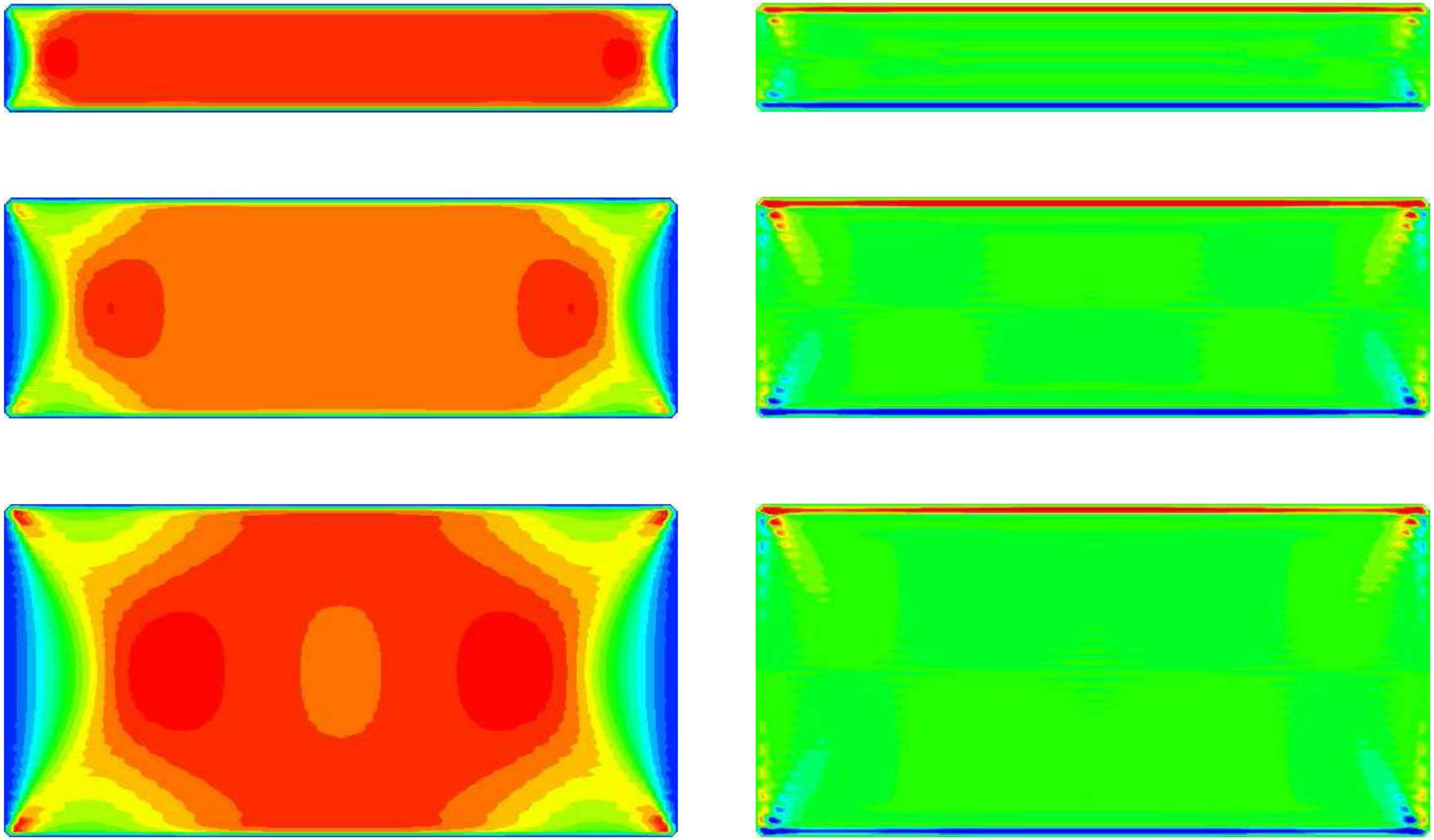
numerical experiments: shear test

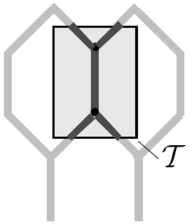


DIEBELS & STEEB, *Proc. R. Soc. Lond. A* **458** (2002)

numerical experiments: shear test

homogenized symmetric shear stress and homogenized couple stress





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numerical experiments: shear test

- analogy to macroscopical Cosserat theory
  - balance of momentum

$$\operatorname{div}\, \mathbf{T} \;=\; \mathbf{0}$$

  - balance of moment of momentum

$$\operatorname{div}\, \bar{\mathbf{M}} + \mathbf{I} \times \mathbf{T} \;=\; \mathbf{0}$$

  - constitutive equations

$$\mathbf{T} \;=\; 2\,\mu\,\bar{\varepsilon}_{\rm sym} \;+\; 2\,\mu_c\,\bar{\varepsilon}_{\rm skw} \;+\; \lambda\,({\rm tr}\;\varepsilon)\,\mathbf{I}$$

$$\bar{\mathbf{M}} \;=\; 2\,\mu_c\,l_c^2\,\kappa$$

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parameter identification based on size effects

- uniaxial tension, one specimen
- simple shear, different specimens of different size

$\longrightarrow$ determination of μ, μ_c, λ, l_c

- validation

  e. g. bending test

validation not successfull!

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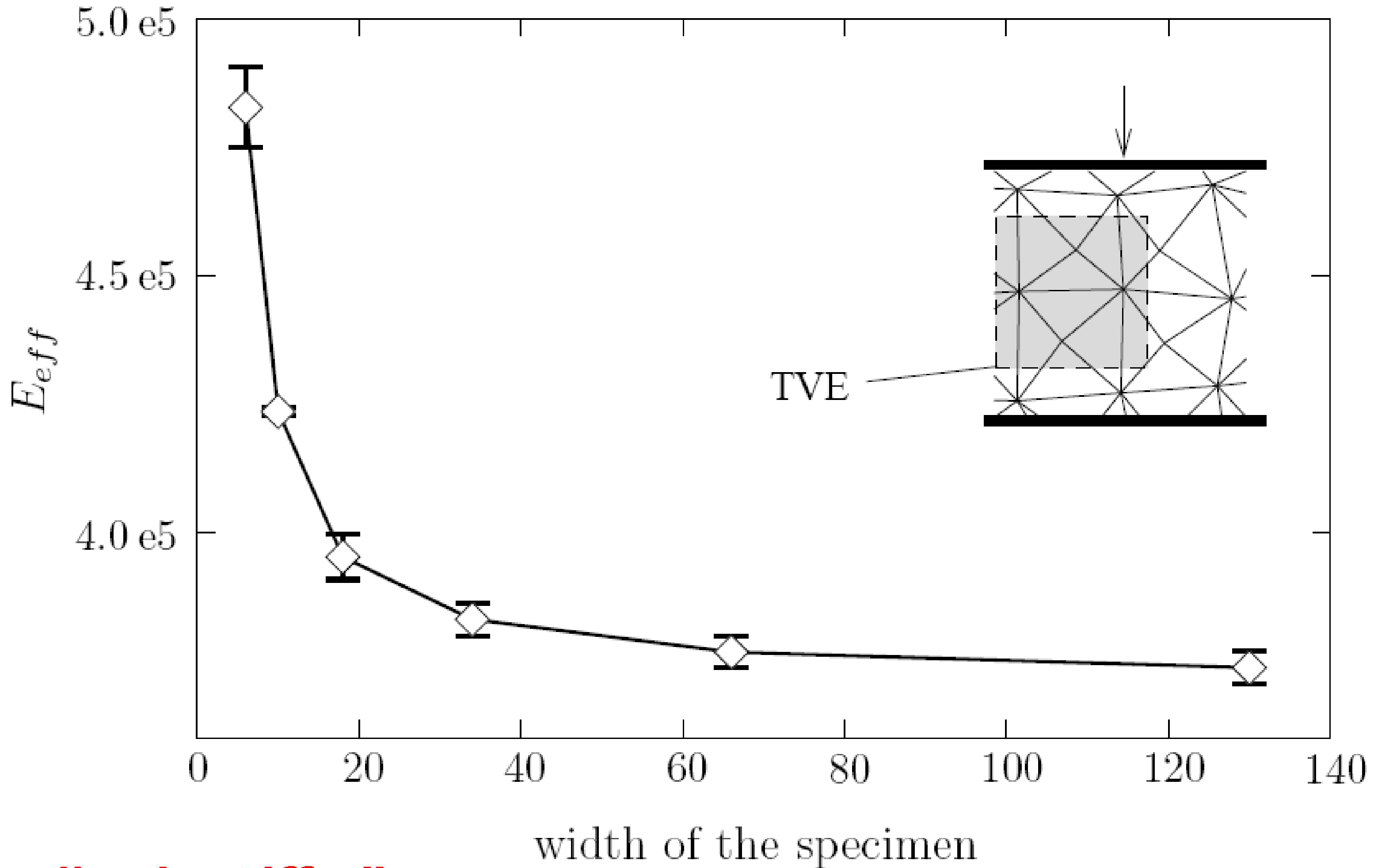
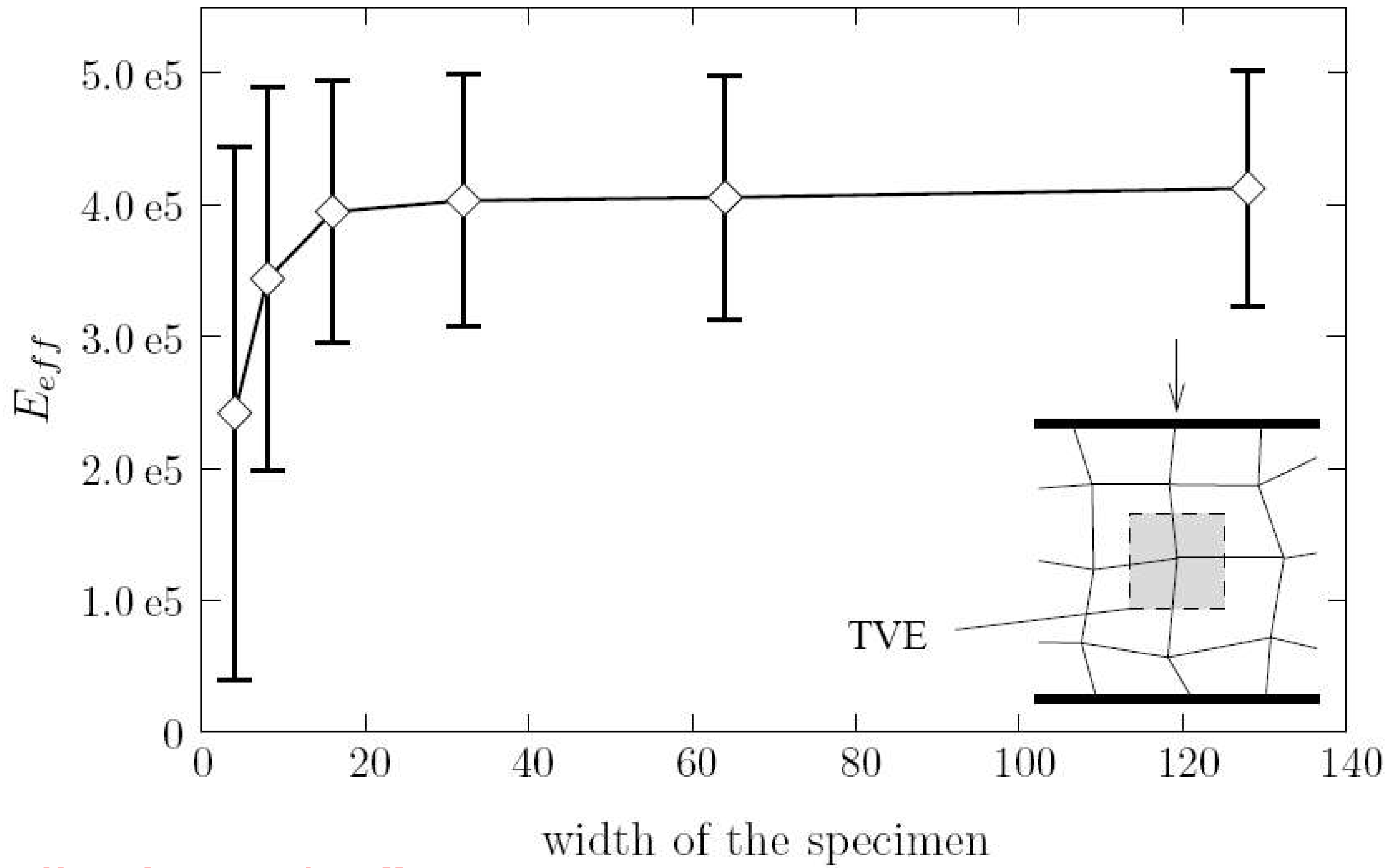
further numerical experiments: uniaxial tension test



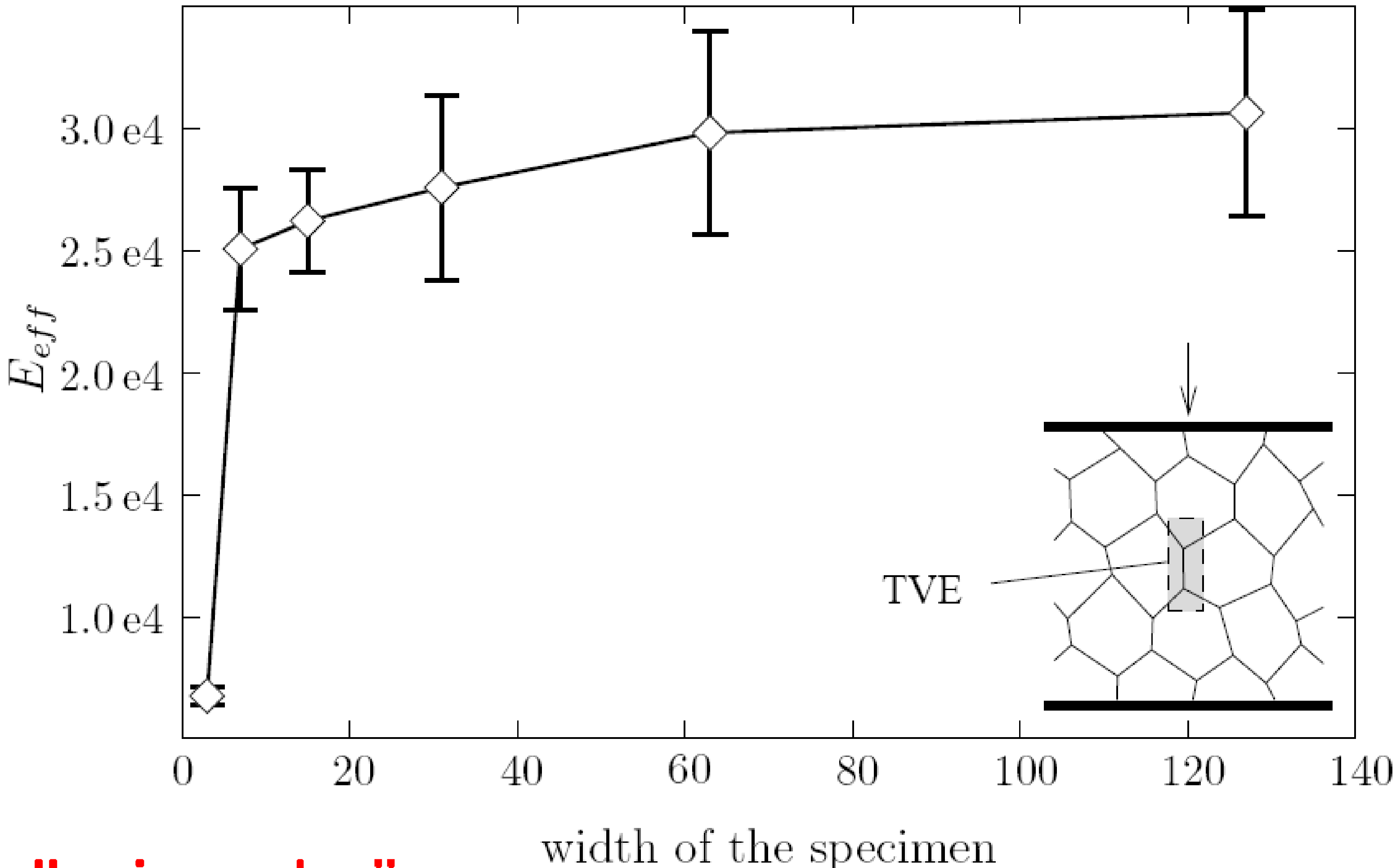
"smaller is stiffer"

further numerical experiments: uniaxial tension test



"smaller is weaker"

numerical experiments: uniaxial tension test



"smaller is weaker"

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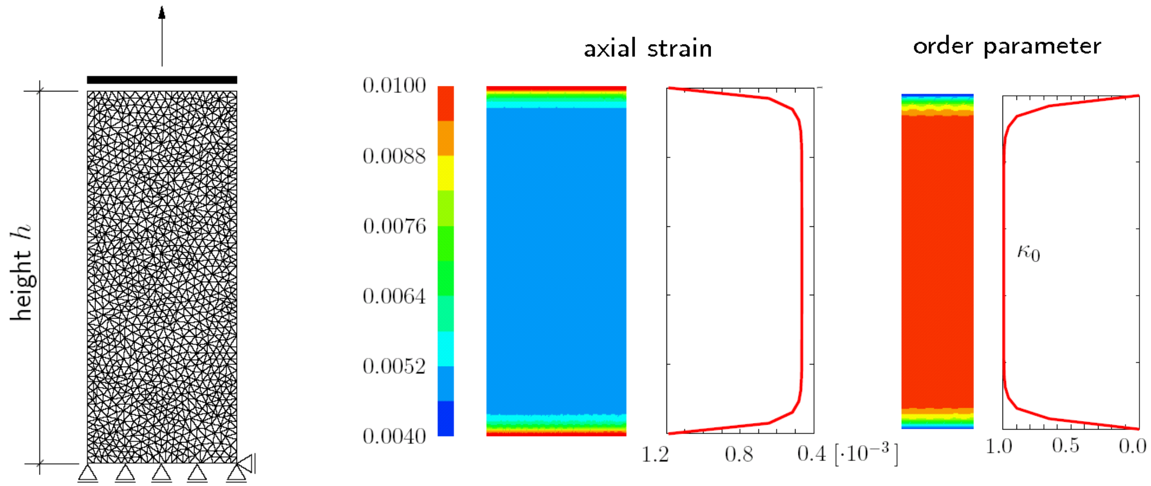
further numerical experiments: uniaxial compression test

- different types of size effects under compression
  - ∗ smaller is stiffer (classical size effect)
  - ∗ smaller is weaker

  depending on local topology

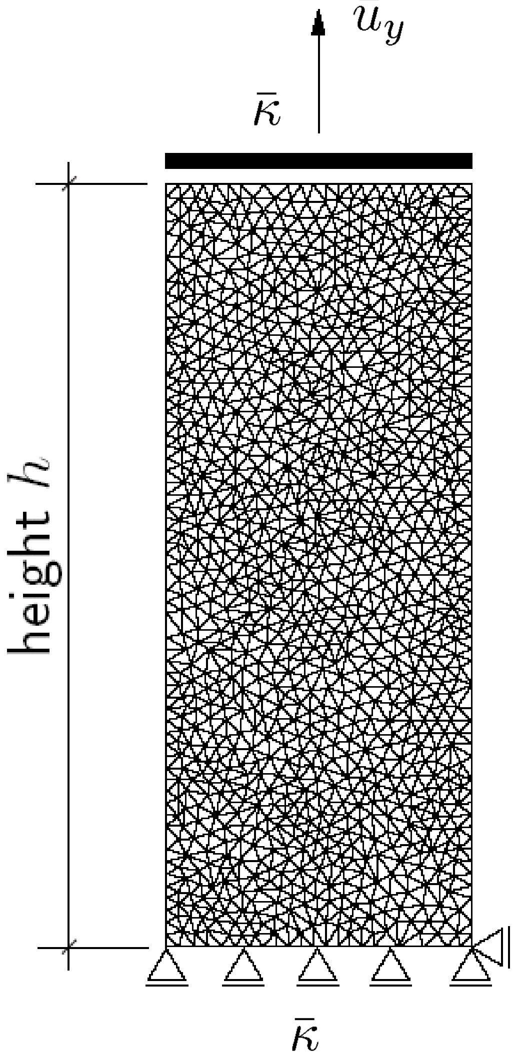
- analogy to continua with order parameter

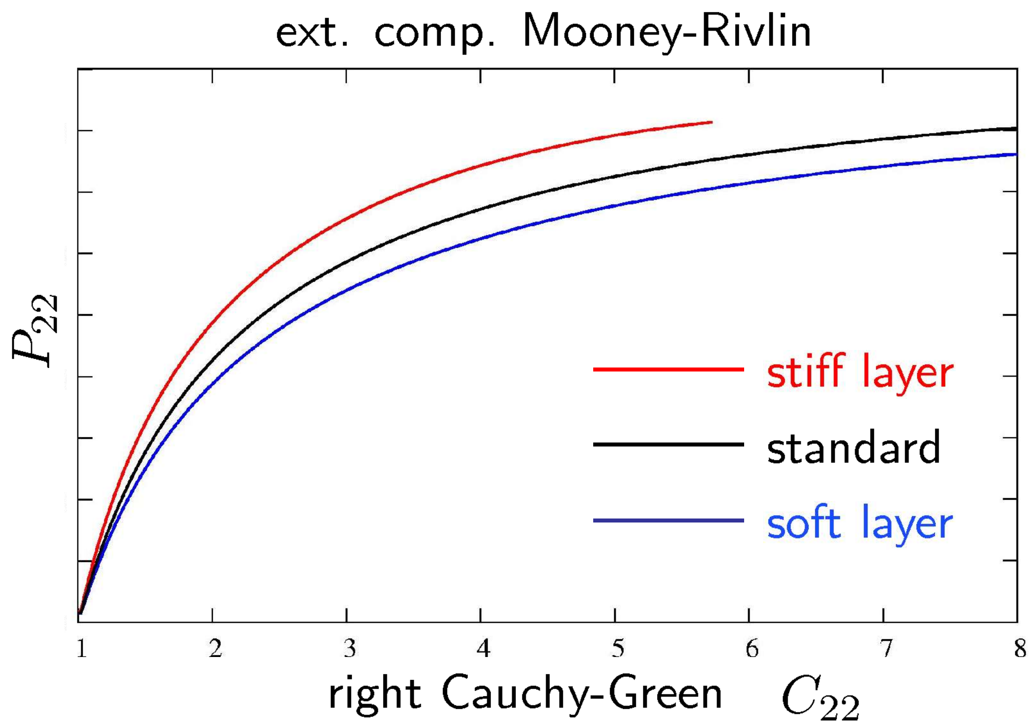
  additional equation of Goodman-Cowin type



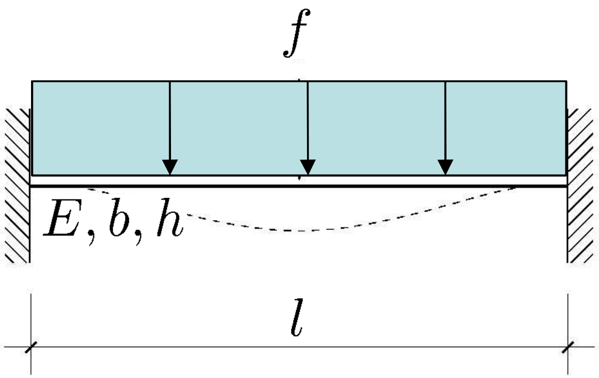
STEEB & DIEBELS, *Int. J. Solids Structures* **41** (2004)

tension experiment: soft & stiff boundary layer





numerical experiments: bending test



maximum deflection:

$$u(x = l/2) = \frac{1}{384} \frac{f\, l^3}{EI}$$

- numerical experiments in analogy to "real" experimental setup

Anderson & Lakes, *Journal of Materials Science* **29** (1994)

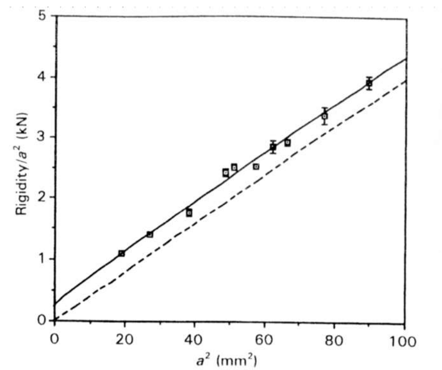


Figure 2 (□) Size-effect results for square, abrasive-machined Rohacell WF300 (width a) in torsion, with Cosserat curve fit for $N^2 = 0.01$. The best-fit curve for this value of N (——) gives $l_t = 0.8$ mm, $l_b = 0.77$ mm and residual error = 3.47 kN^2. The classical curve (– – –) has a residual error of 50.5 kN^2.

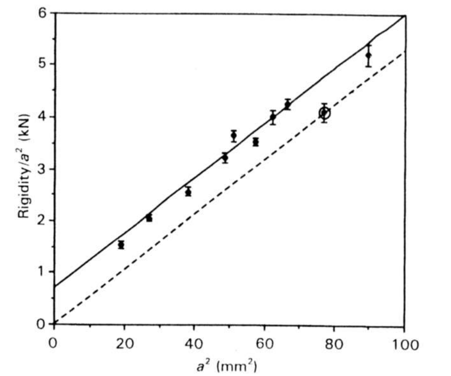
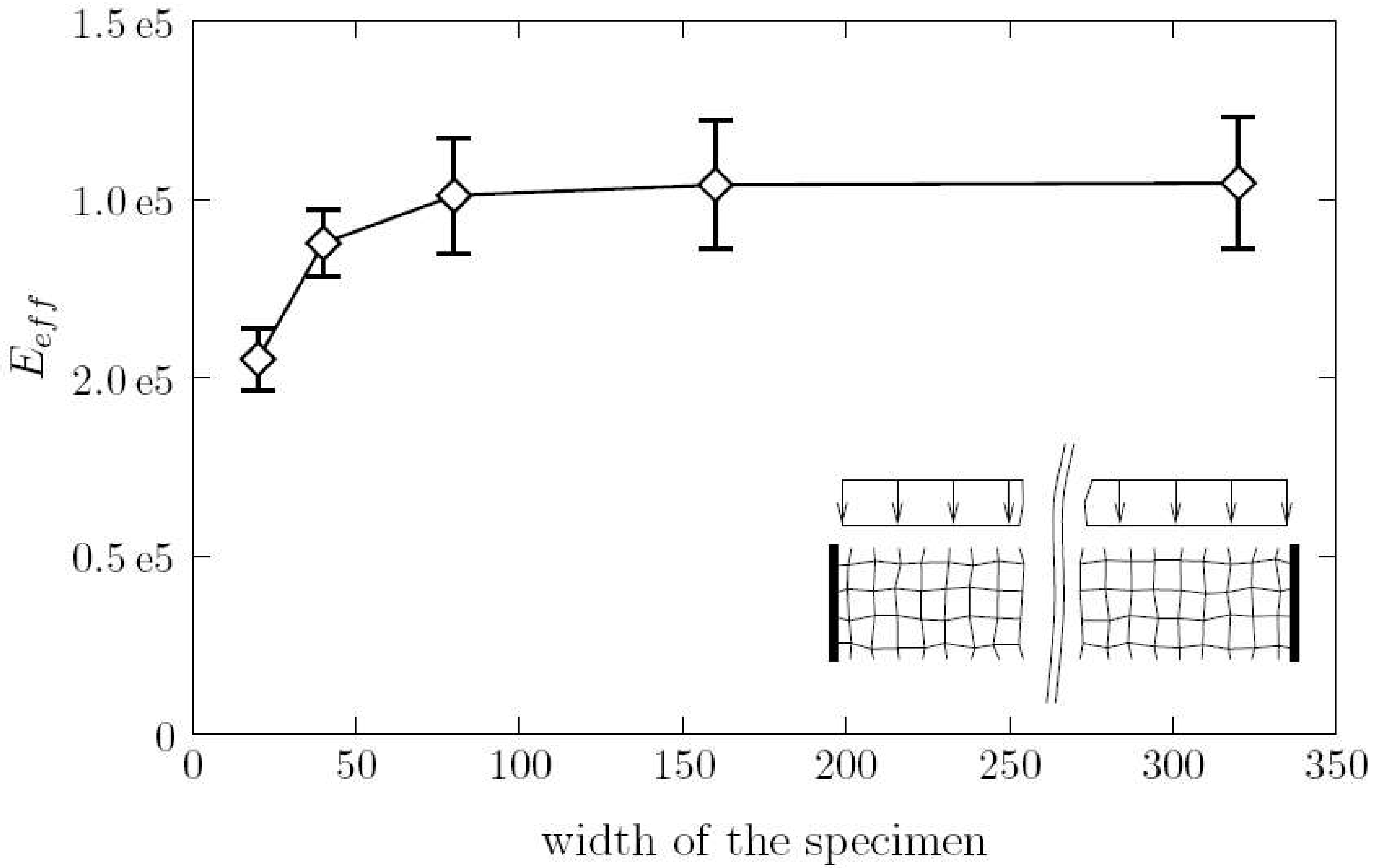


Figure 3 (●) Size-effect results for square, abrasive-machined Rohacell WF300 (width a) in bending. The best-fit Cosserat curve (——) gives $E = 637$ MPa, $l_b = 0.78$ mm, $\nu = 0.13$ and residual error = 34.6 kN^2. The classical curve (– – –) has a residual error of 550 kN^2. One point (circled) is a statistical outlier not used in residual calculations.

numerical experiments: bending test



"smaller is weaker"

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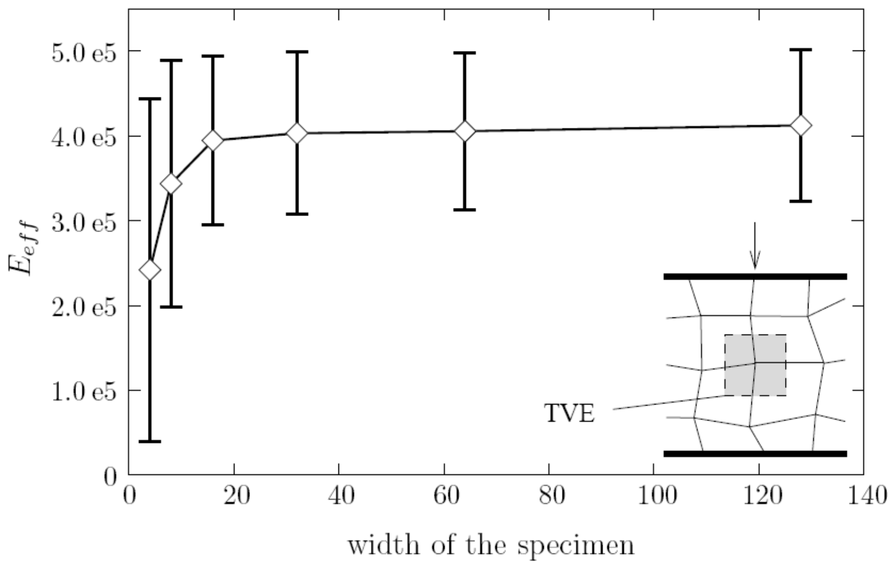
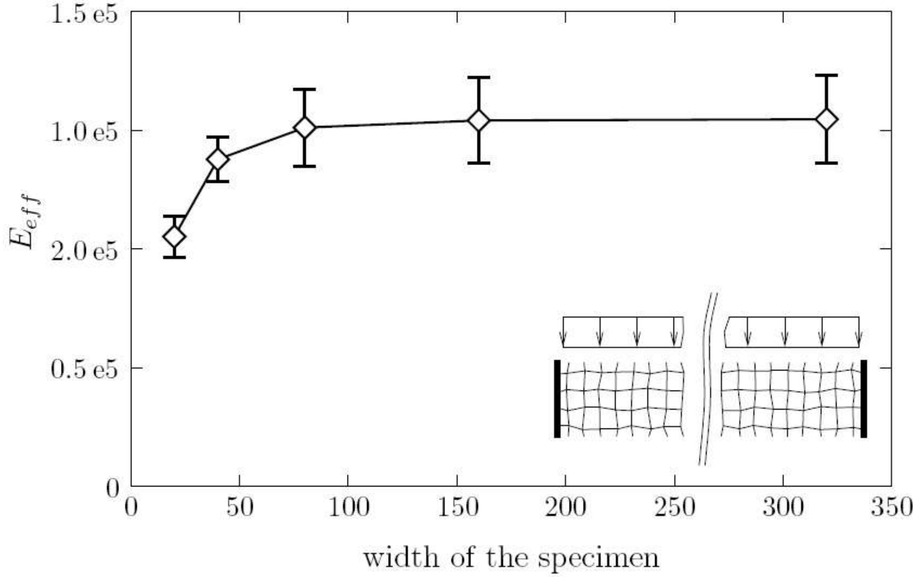
numerical experiments: bending test

effective modulus (tension)



effective modulus (bending)



microstructured material does not behave according to beam theory

finite deformations

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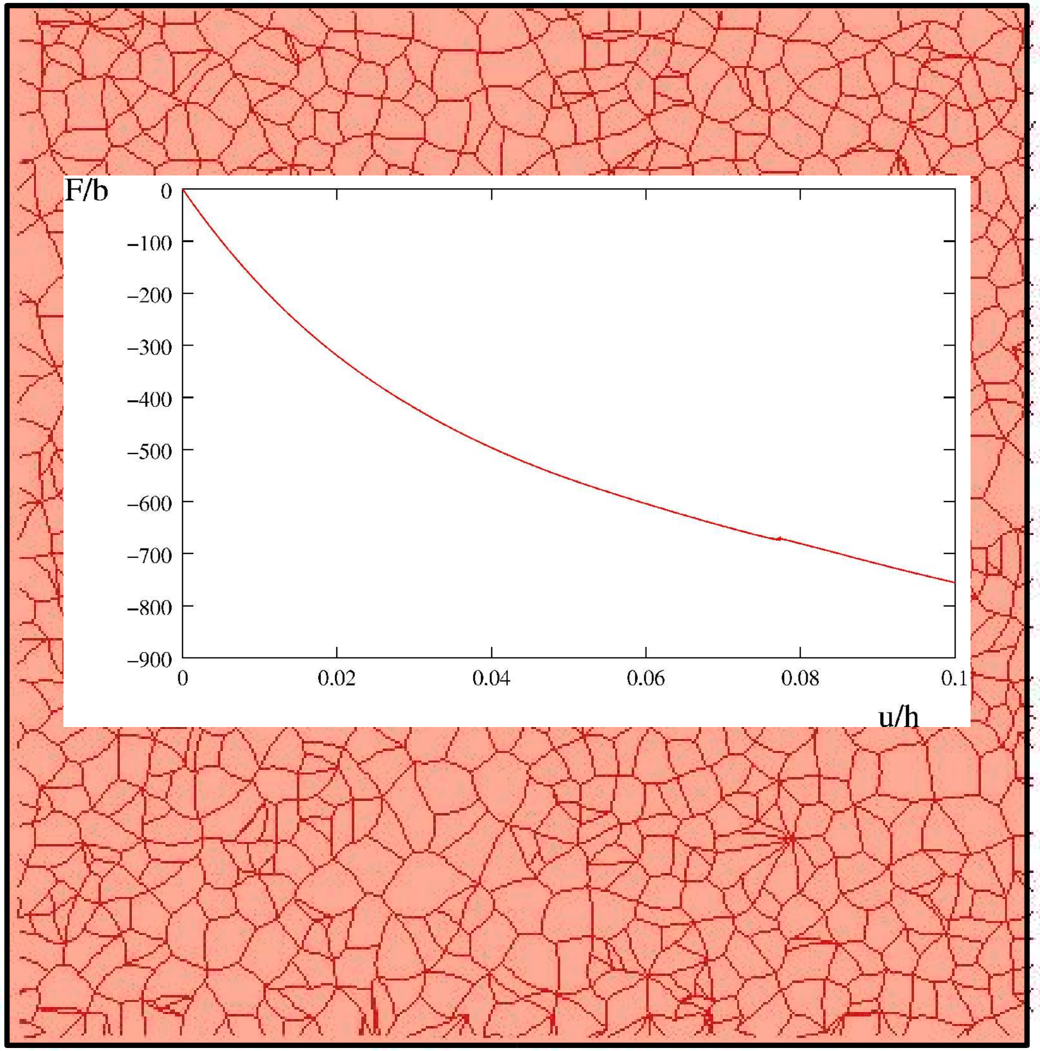
numerical experiments: compression test



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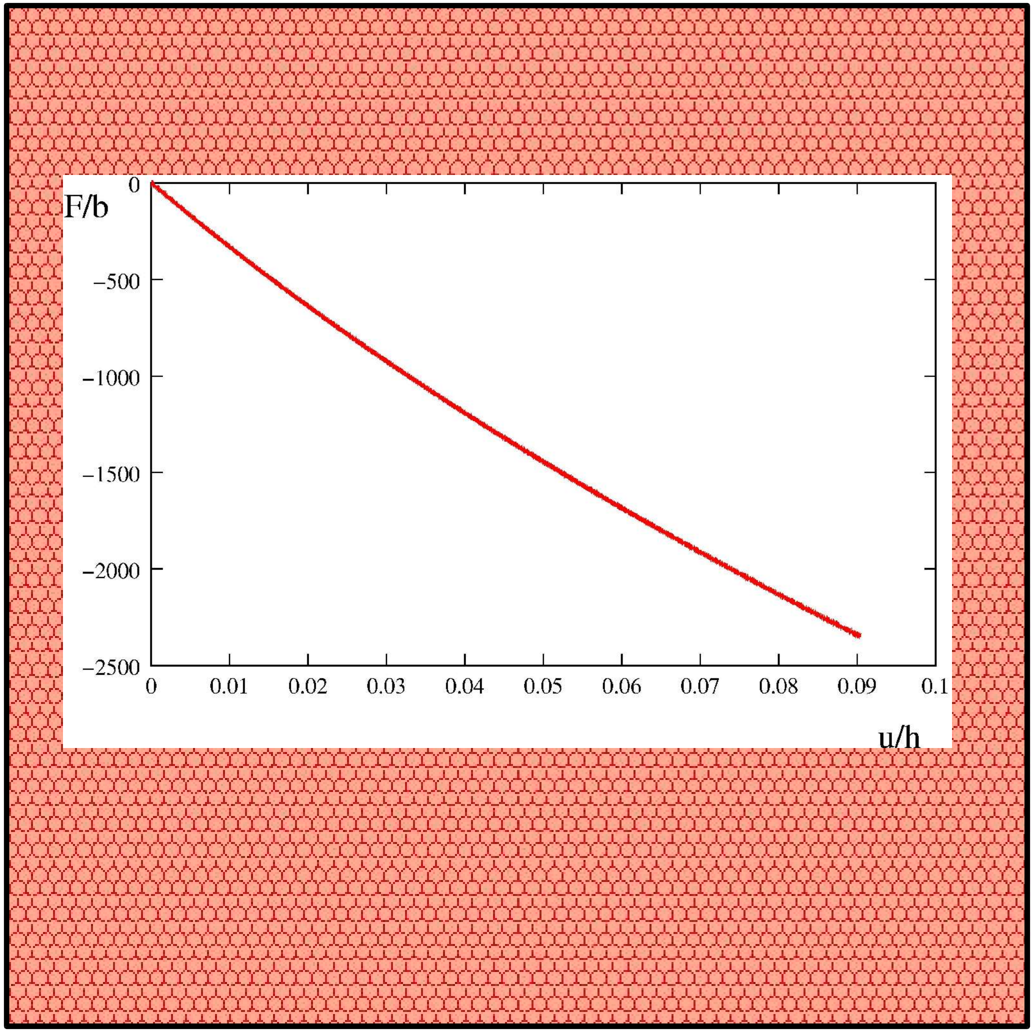
numerical experiments: compression test



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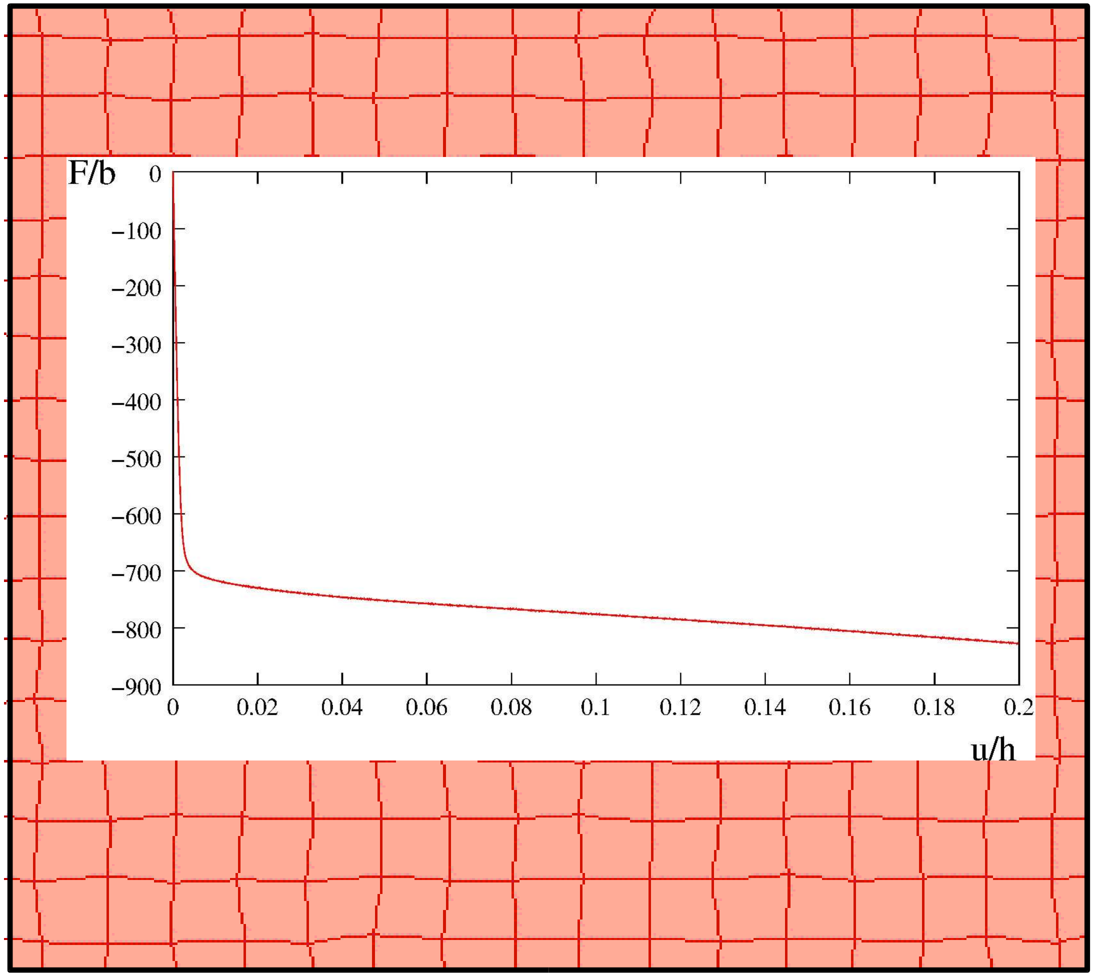
numerical experiments: compression test



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numerical experiments: compression test

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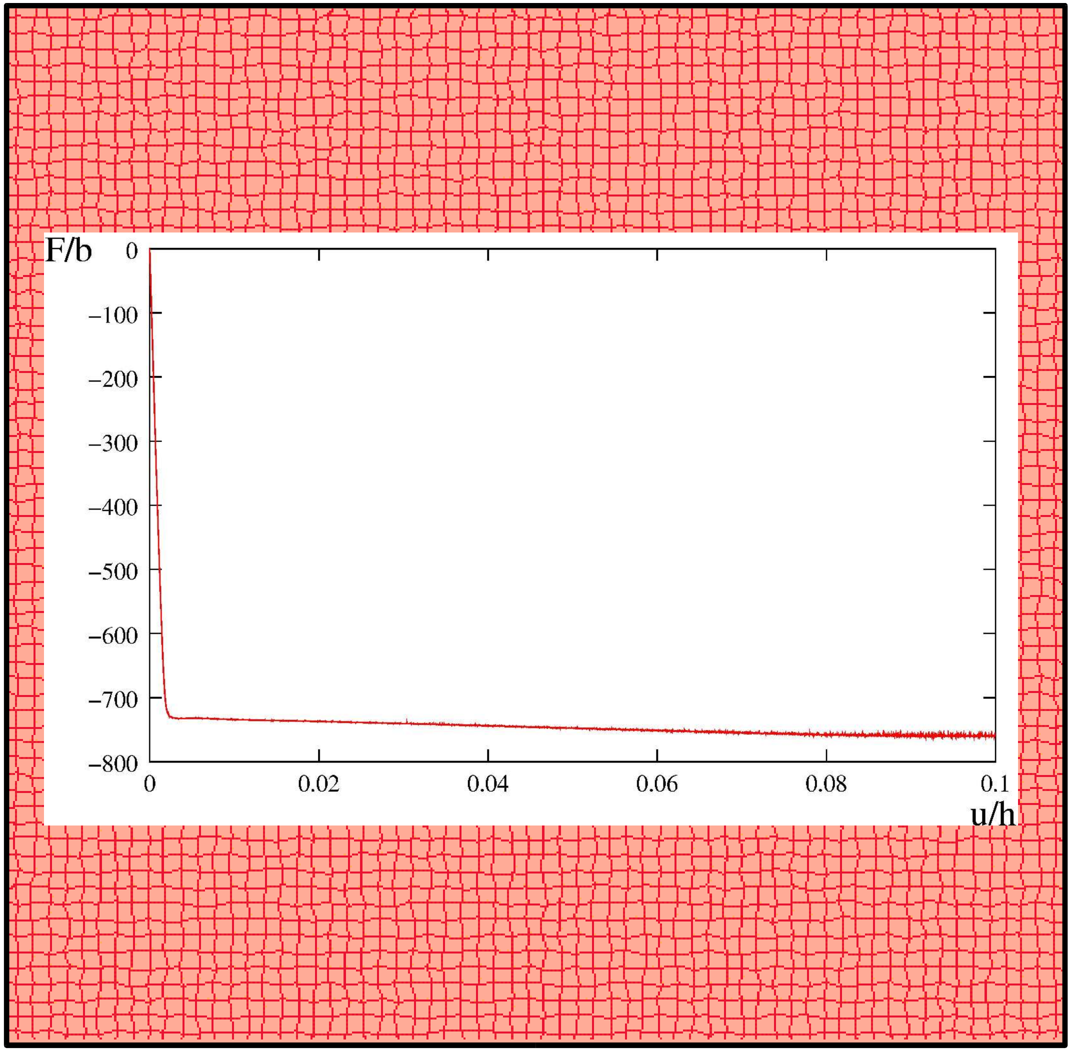

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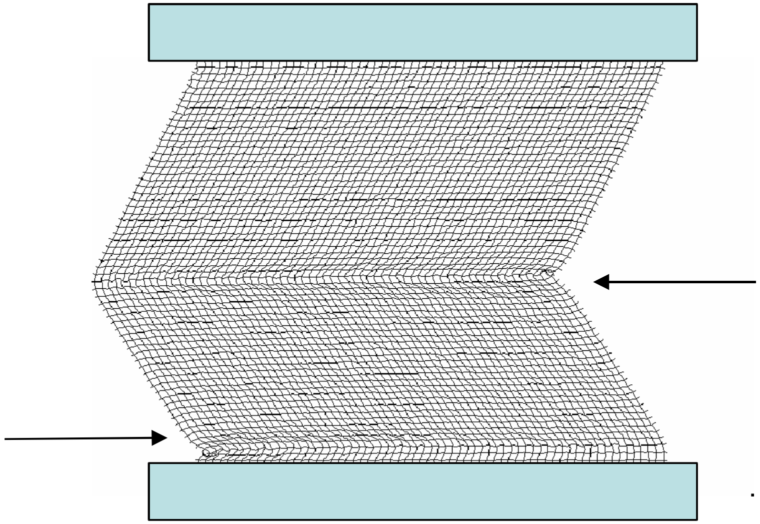
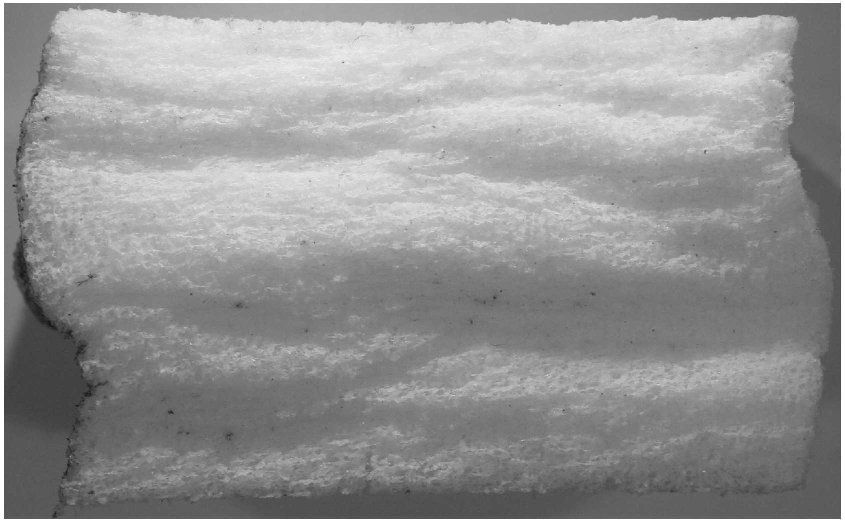
LTM

numerical experiments: compression test





higher order instability modes

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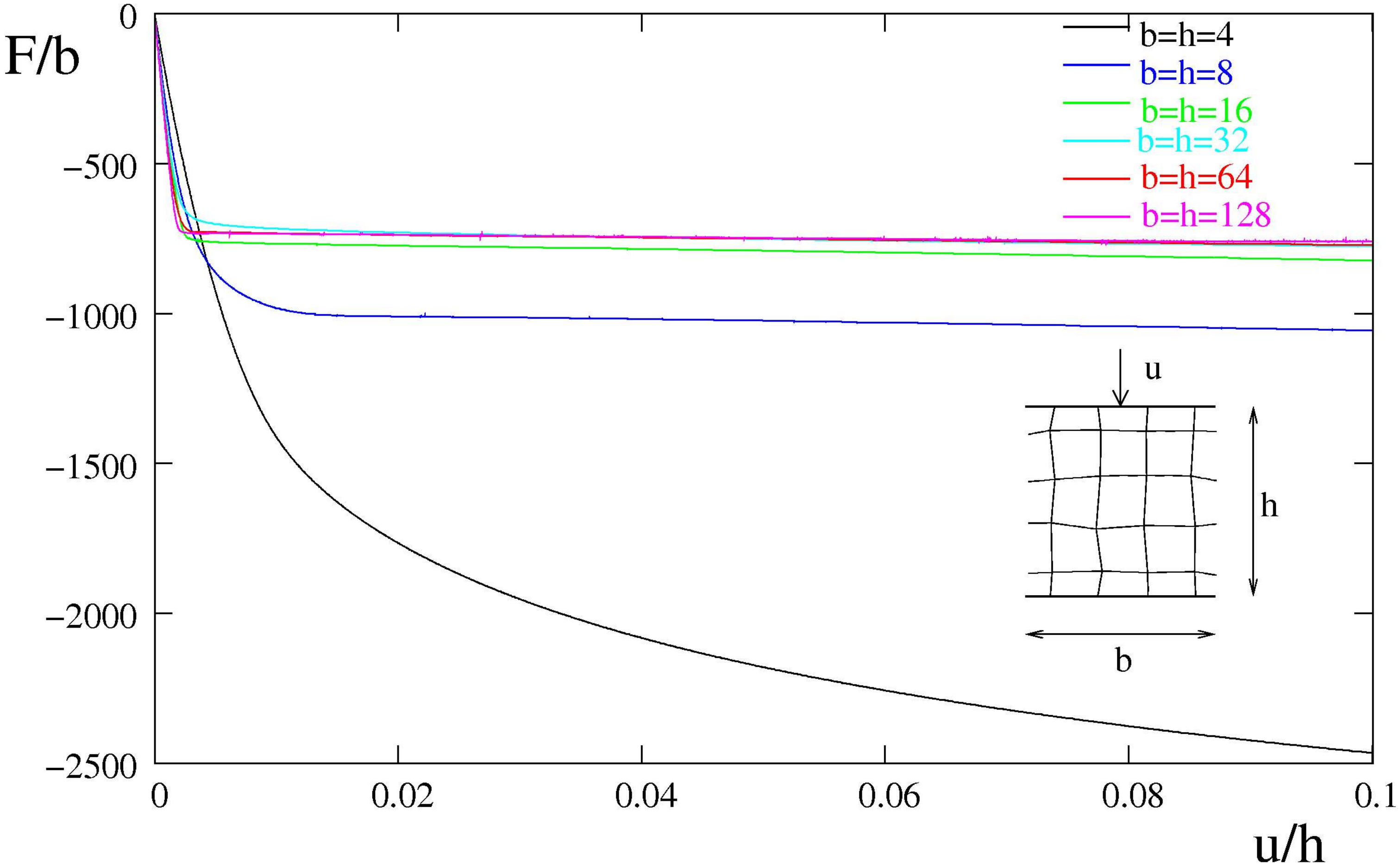
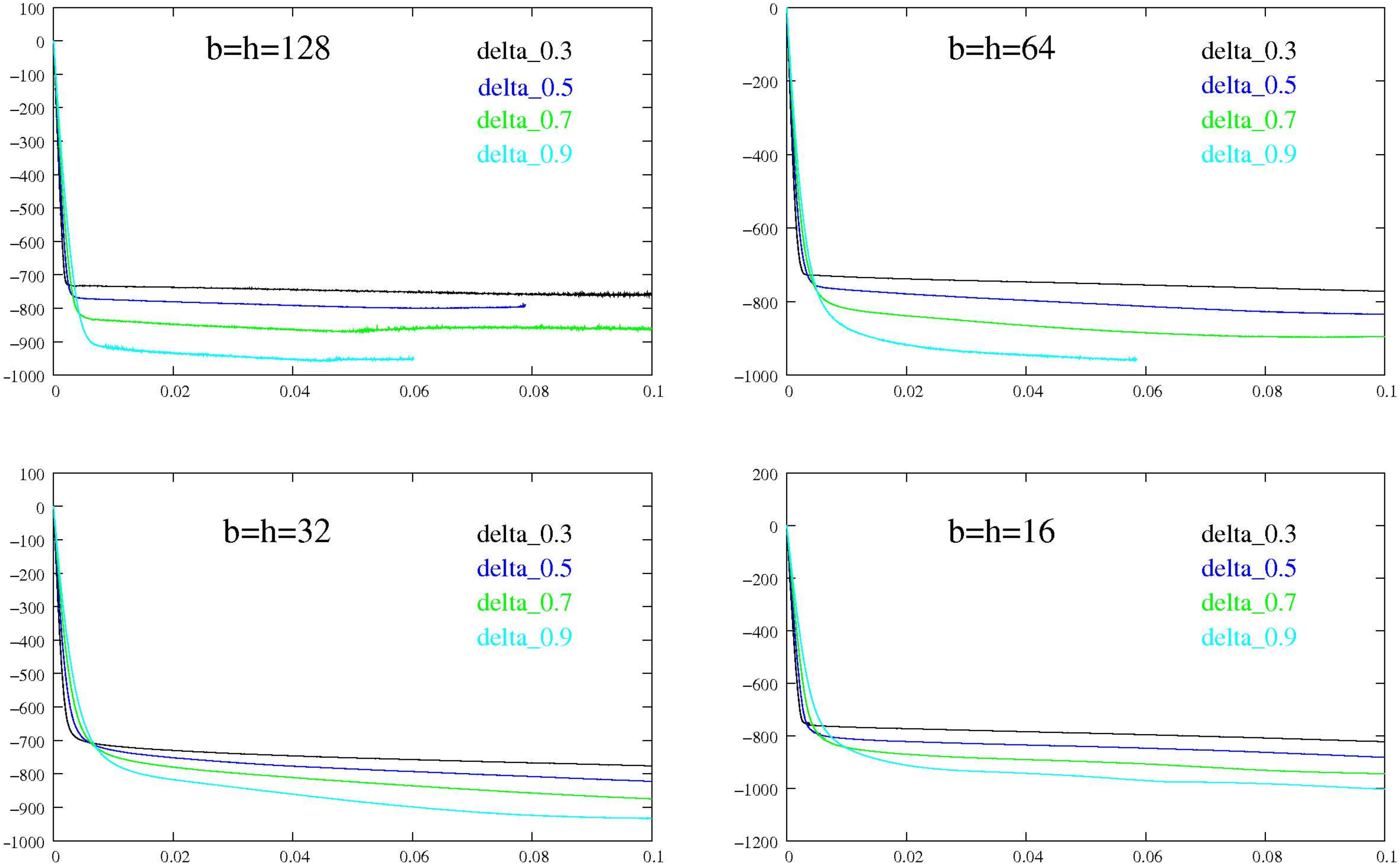
stress-deformation curves



stress-deformation curves



discussion:

- size effects observed in
  - * shear
  - * compression
  - * bending
- size effects depending on topology
  - * smaller is stiffer
  - * smaller is weaker
- inconsistent moduli in compression and bending
- higher instability modes possible

discussion:

- Cosserat continuum (MMM-continua)
  - − only stiff boundary layers
  - + simple interpretation in terms of microstructure
- non-local continua, gradient continua
  - − no interpretation in terms of microstructure
  - + weak and stiff boundary layers
- generalized continua with (scalar) order parameter
  - − no interpretation in terms of microstructure
  - + weak and stiff boundary layers