PAMM · Proc. Appl. Math. Mech. **11**, 209 – 210 (2011) / **DOI** 10.1002/pamm.201110096

Consistent time integration for transient three-dimensional contact using the NTS-method

Marlon Franke[1,*], **Christian Hesch**[1], and **Peter Betsch**[1]

[1] Chair of Computational Mechanics, University of Siegen, Paul-Bonatz-Str. 9-11, D-57068 Siegen, Germany

The present work aims to investigate stable time integrators for large deformation contact problems within the framework of the well known node-to-segment (NTS)-method. For this kind of problem, standard time integrators fail to conserve the total energy of the system. To remedy this drawback, we combine a mixed method with the concept of a discrete gradient applied to the aforementioned NTS-method. In the context of nonlinear elastodynamics stable integrators for ordinary differential equations have been extensively developed and investigated during the last two decades. For contact problems, energy consistent integrators have been developed for the NTS-method (see e.g. Ref. [1]).

1 Continuum and contact mechanics

To describe large deformation contact problems, we consider k bodies $\mathcal{B}^{(i)} \subset \mathbb{R}^3$, $i \in [1, ..., k]$. In Fig. 1 both the reference and the current configuration are displayed, where we limit ourself to a two body contact problem without restricting the generality. The surfaces of both bodies are subdivided into the Dirichlet boundary Γ_u, the Neumann boundary Γ_σ and the contact surface Γ_c. We assume that the boundaries of each body satisfy the conditions:

$$\partial\mathcal{B}^{(i)} = \Gamma_\sigma^{(i)} \cup \Gamma_c^{(i)} \cup \Gamma_u^{(i)} \quad \text{and} \quad \Gamma_\sigma^{(i)} \cap \Gamma_c^{(i)} = \Gamma_\sigma^{(i)} \cap \Gamma_u^{(i)} = \Gamma_c^{(i)} \cap \Gamma_u^{(i)} = \emptyset$$

For the virtual contact work of both bodies we obtain:

$$\sum_{i=1}^{2} G^{(i)}\left(\boldsymbol{\varphi}, \delta\boldsymbol{\varphi}\right) = 0 = \sum_{i=1}^{2} \left(G^{(i),dyn} + G^{(i),int} + G^{(i),ext} + G^{(i),c} \right) \tag{1}$$

The first term in equation (1) on the right hand side is related to the dynamic virtual work, the second and third term represent the virtual work of the internal and external forces and the last term denotes the contact contribution to the virtual work.

2 Discretization in space and time

The bodies are discretized using displacement-based finite elements. In this connection we use isoparametric finite element interpolations with shape functions for an eight-node tri-linear brick element. For the description of the virtual contact work

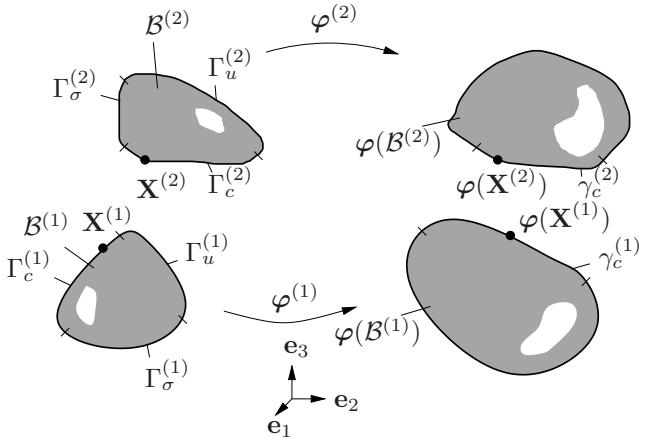

Fig. 1 Configurations due to two body contact.

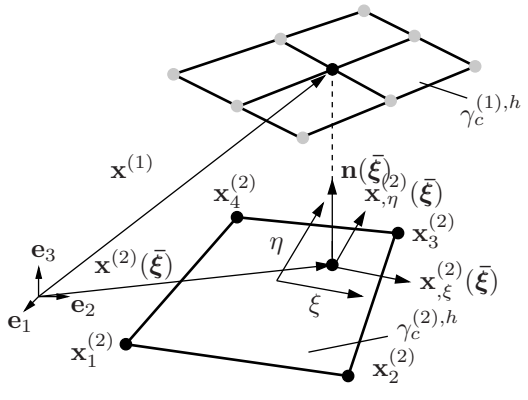

Fig. 2 Five node NTS-element.

we use the well known NTS-method (see e.g. the textbooks Ref. [4, 5]). The discrete gap function g_{Ns} denotes the closest point projection from the slave point $\mathbf{x}^{(1)}$ to the corresponding master surface $\mathbf{x}^{(2)}(\bar{\boldsymbol{\xi}}) = \sum_{K=1}^{4} \hat{N}_K(\bar{\boldsymbol{\xi}})\, \mathbf{x}_K^{(2)}$. For

* Corresponding author: Email franke@imr.mb.uni-siegen.de, phone +49 271 740 4637, fax +49 271 740 2436

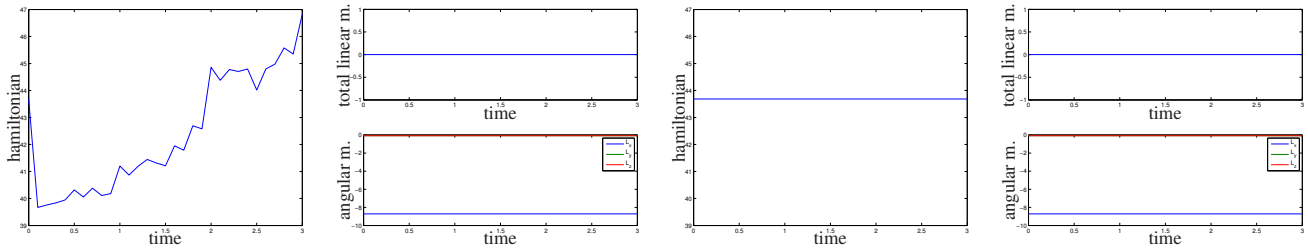

Fig. 3 Energy, total linear momentum and angular momentum due to midpoint rule (three figures on the left) and energy-momentum scheme (three figures on the right), respectively.

the frictionless case we need to fulfill the Kuhn Tucker complementary conditions for the normal direction. To enforce the Kuhn-Tucker conditions, there exist different methods. We focus here on the Lagrange multiplier method where we use an active-set strategy in order to incorporate the inequalities due to the Kuhn-Tucker complementary conditions. To remedy the drawback of standard integrators, which fail to conserve the energy of a conservative system and consequently get unstable, we combine a mixed method with the concept of a discrete gradient applied to the aforementioned NTS-method. To facilitate the design of such energy-momentum schemes we introduce a mixed formulation and apply the concept of a discrete gradient. Finally we obtain the completely discrete differential-algebraic equations (DAE):

$$
\begin{aligned}
\mathbf{x}_{n+1} - \mathbf{x}_n - h\,\mathbf{v}_{n+\frac{1}{2}} &= \mathbf{0} \\
\mathbf{M}\,(\mathbf{v}_{n+1} - \mathbf{v}_n) + h\,\overline{\nabla}_{\mathbf{x}} V(\mathbf{x}_n, \mathbf{x}_{n+1}) & \\
+h \sum_{s=1}^{n_c} \left(\mathrm{D}_1\, \boldsymbol{\pi}(\mathbf{x}_{s,n+\frac{1}{2}}, \mathbf{d}_{s,n+\frac{1}{2}}, \mathbf{f}_{s,n+\frac{1}{2}}) \right)^{\mathrm{T}} \overline{\overline{\nabla}}_{\boldsymbol{\pi}} \tilde{\mathbf{g}}_s(\boldsymbol{\pi}_n, \boldsymbol{\pi}_{n+1}) \cdot \boldsymbol{\lambda}_{s,n+1} &= \mathbf{0} \\
\sum_{s=1}^{n_c} \left(\mathrm{D}_2\, \boldsymbol{\pi}(\mathbf{x}_{s,n+\frac{1}{2}}, \mathbf{d}_{s,n+\frac{1}{2}}, \mathbf{f}_{s,n+\frac{1}{2}}) \right)^{\mathrm{T}} \overline{\overline{\nabla}}_{\boldsymbol{\pi}} \tilde{\mathbf{g}}_s(\boldsymbol{\pi}_n, \boldsymbol{\pi}_{n+1}) \cdot \boldsymbol{\lambda}_{s,n+1} &= \mathbf{0} \\
\sum_{s=1}^{n_c} \left(\mathrm{D}_3\, \boldsymbol{\pi}(\mathbf{x}_{s,n+\frac{1}{2}}, \mathbf{d}_{s,n+\frac{1}{2}}, \mathbf{f}_{s,n+\frac{1}{2}}) \right)^{\mathrm{T}} \overline{\overline{\nabla}}_{\boldsymbol{\pi}} \tilde{\mathbf{g}}_s(\boldsymbol{\pi}_n, \boldsymbol{\pi}_{n+1}) \cdot \boldsymbol{\lambda}_{s,n+1} &= \mathbf{0} \\
\begin{bmatrix} \tilde{\mathbf{g}}_1(\boldsymbol{\pi}(\mathbf{x}_{1,n+1}, \mathbf{d}_{1,n+1}, \mathbf{f}_{1,n+1})) \\ \vdots \\ \tilde{\mathbf{g}}_{n_c}(\boldsymbol{\pi}(\mathbf{x}_{n_c,n+\frac{1}{2}}, \mathbf{d}_{n_c,n+\frac{1}{2}}, \mathbf{f}_{n_c,n+\frac{1}{2}})) \end{bmatrix} &= \mathbf{0}
\end{aligned}
$$

where $s = 1, ..., n_c$ denotes the sth contact element (see Fig. 2). Here a discrete gradient applied to the internal energy is used (for further informations see Ref. [2]). In addition, a discrete gradient applied on the contact constraints is used (see Ref. [3]). The advantage of this energy-momentum scheme is that besides the algorithmic consistency of the angular-momentum the total energy of the system is conserved, leading to a remarkably stable time integration scheme.

3 Numerical example

We compare the midpoint rule with the proposed energy-momentum scheme with a time step size of $h = 0.1$. No external forces and torques are acting on the bodies, so we are dealing with a conservative system, which means that the basic properties of the bodies are conserved. But as illustrated in Fig. 3, it becomes obvious that the energy-momentum scheme conserves all quantities whereas the midpoint rule fails to conserve the total energy of the system.

References

[1] F. Armero and E. Petöcz, Formulation and analysis of conserving algorithms for frictionless dynamic contact/impact problems, *Comput. Methods Appl. Mech. Engrg.*, **158**, 269–300, 1998.
[2] P. Betsch and P. Steinmann, Conservation properties of a time FE method. Part II: Timestepping schemes for nonlinear elastodynamics. *Int. J. Numer. Meth. Engng*, **50**, 1931–1955, 2001.
[3] C. Hesch and P. Betsch, Transient 3d contact problems – NTS method: Mixed methods and conserving integration, *Computational Mechanics*, DOI: 10.1007/s00466-011-0597-2
[4] T.A. Laursen, *Computational contact and impact mechanics*. Springer-Verlag, 2002.
[5] P. Wriggers, *Computational contact mechanics*. Springer-Verlag, 2nd edition, 2006.

